EFCE
7/7/08
11:30 a.m.

Recovery Before Redemption? A Theory of Delays in Sovereign Debt Renegotiations

David Benjamin*
University of Southampton

Mark L. J. Wright*
University of California, Los Angeles

May 28, 2008

Abstract

In this paper, we present evidence from a new database to show that negotiations to restructure sovereign debts are lengthy and ineffective at reducing indebtedness, taking more than eight years to complete, resulting in creditor losses of more than forty per-cent, and leaving debtor countries with debt levels forty per-cent higher (scaled by GDP) than when they entered default. We also document the economic circumstances that lead to especially protracted defaults. We then present a theory of sovereign debt renegotiation that can account for these empirical findings. In the theory, delays arise due to the same commitment issues that lead to default: as the debtor's ability to share future surplus created by a debt restructuring is limited by future default risk, the debtor and creditor find it privately optimal to delay restructuring until future default risk is low, even when delay may be socially inefficient.

*We thank Vivian Yue, and seminar participants at the University of California, Los Angeles, the University of Texas at Austin, and the 2007 Society for Economic Dynamics meetings in Prague for comments. Further comments welcome.

1. INTRODUCTION

Negotiations to restructure sovereign debts are time consuming, taking more than seven years, on average, to complete. In this paper, we present evidence from a new database covering almost one-hundred recent sovereign default renegotiation that shows that negotiations are also ineffective in both repaying creditors and reducing the debt burden countries face. Creditors suffer a write-down (or *haircut*) of more than forty per-cent, on average, while defaulting countries exit default with a debt level that is forty per-cent higher, scaled by gross domestic product (GDP), than when they entered default. We also show that these events are related: long delays in restructuring are associated with greater haircuts, while both the amount of delay and the size of the haircut are correlated with economic conditions at the time of default. We then present a theory of sovereign debt renegotiation in which outcomes are driven by both fluctuations in domestic economic conditions, as well as changes in creditor and debtor bargaining power. In the theory, delays arise because the debtor's ability to share future surplus created by a debt restructuring is limited by future default risk. Both the debtor and creditor find it privately optimal to delay restructuring until future default risk is low, even though this delay is socially inefficient. We conclude by demonstrating that a calibrated version of the theory accounts for our empirical findings.

We begin by presenting our database on sovereign debt renegotiation outcomes. Drawn from a variety of sources, the database covers ninety defaults by seventy-three countries that were settled during the period 1989 to 2006 and contains data on the occurrence of default, the outcome of negotiations, indicators of economic activity, as well as measures of a country's foreign investment position. We use these data to establish the following six facts about the process of sovereign debt renegotiation, of which four are new. First, and as has been pointed out by many other authors, *sovereign defaults are time consuming to resolve*, taking over eight years on average between the initial announcement of default to majority acceptance of a settlement offer. Second, *creditor losses (or* haircuts*) are substantial*, with the average creditor experiencing a reduction in the value of their claim of forty-four per-cent. Third, *default resolution is associated with increased country indebtedness*, with the average defaulting country having a debt to GDP ratio of fifty-two per-cent in the year prior to default, and exiting default with a debt to GDP ratio of almost seventy-two per-cent. Fourth, *longer defaults are associated with larger haircuts*, with a correlation between the length of the renegotiation process and the size of the creditor haircut of two-thirds. Fifth, there is a modest negative relationship between economic conditions in the years surrounding default and renegotiation outcomes: *larger output declines in the year of default are associated with longer defaults and larger haircuts*, with correlation coefficients around -0.20. Lastly, and as was previously demonstrated by Tomz and Wright (2007), *defaults are somewhat more likely to occur when output is below trend, and settlements tend to occur when output has returned partially to trend*, in a sense that we make precise below.

We then present our theory of sovereign debt renegotiation and evaluate its ability to account for these observations. In our theory, a sovereign country borrows from a competitive group of international lenders using defaultable, but otherwise state non-contingent, bonds in order to smooth consumption and tilt its consumption profile. The country may choose to default at any time, in which case it must renegotiate its debts before it is able to reaccess international capital markets. As a consequence, default and debt renegotiation serve to partially ameliorate the lack of state contingent debt.

Debt restructuring negotiations take the form of a non-cooperative bargaining game with complete information. Creditors are assumed to be able to perfectly coordinate so that no delay occurs due to collective action problems. The debtor and (representative) creditor randomly alternate in their ability to propose a bargaining outcome, with changes in the probability of making future proposals serving to capture changes in bargaining power. Bargaining outcomes include both a transfer of current resources, as well as a new issue of debt, with the creditor's payoff depending on the level of transfers and on the market value of these new debts. At any time, the debtor country has the "outside option" to repay the defaulted debt in full.

Delays in bargaining are socially costly *ex post*, as the debtor country is excluded from international capital markets while in default and hence potential gains from trade remain unexploited. Sub-game perfect equilibria for our theory are always optimal *ex post*, from the perspective of the bargaining parties. Nonetheless some sub-game perfect equilibria feature delays. These delays are a result of the same commitment problem that led to default in the first place. Most of the surplus generated by a debt renegotiation accrues after the negotiations are completed. Although the debtor country would like to commit to share this future surplus in any renegotiation, it may default on any promise to do so. As a result, new debt issues as part of a renegotiation may have little value and it can be in the best interests of the parties to delay completing the renegotiation until the value of new debt issues are higher. Debt is more valuable when the risk of future default is lower and the expected settlement from any future debt renegotiation to the creditors is higher. In turn, both the risk of default and the expected future settlement are functions of expectations surrounding future income levels and bargaining power.

We then show that a calibrated version of our theory is able to account for the facts outlined above. The key to our calibration is the choice of a bargaining regime. Countries default in one of two circumstances. A country may default because output is low, debt is high, and there is no other means of smoothing consumption. Alternatively, default also may occur when the debtor's bargaining power is high. When we calibrate fluctuations in bargaining power to match aspects of the relationship between default and output in the data, we find that the model produces defaults lasting five and one-half years on average, as both the country and the creditor wait until economic conditions improve, or debtor bargaining power is reduced, both of which increase the value of any new debt issues, before agreeing to a proposed debt renegotiation. We also reproduce many key outcomes in the model, including the correlation between large haircuts and long defaults and the tendency of countries to leave default with more debt than when they entered default. Default tends to occur slightly more often when output is below trend, and agreement tends to be reached when output has returned halfway to its trend level. The tendency for the parties to delay agreement until output has moved partially back to trend produces a positive relationship between the size of the initial output decline (a positive number) and the length of the default. As output is still relatively low at the time the renegotiation is concluded, the settlement includes some new borrowing and a current transfer of resources to the country. The lengthy delays, combined with new loans to the country, produce substantial creditor haircuts, with the largest haircuts being associated with the largest delays.

Moreover, we argue that our theory is able to explain one other key observation about sovereign defaults: *defaults are highly correlated accross countries.* Previous authors have argued that this is as a result of synchronization in debtor country business cycles, but this view is inconsistent with the modest relationship between default and output observed in the data. Our theory suggests that changes in debtor bargaining power, associated with changes in creditor country legislation or the policies of creditor country governments and supranational institutions, is capable of explaining synchronized defaults. For example, our model suggests that US government and IMF intervention in the 1980s may have contributed to a worsening of the Latin American Debt Crisis, while the rise of litigious, prone to holding-out, creditors in the 1990s may have contributed to it's end.

Our paper makes contributions to four distinct literatures. First, relative to the empirical literature on default, in constructing our database of sovereign debt renegotiation outcomes we follow Cline (1995) in extending the methods of the World Bank (1993) in calculating creditor losses for our sample of countries. This methodology has been criticized by Sturzenegger and Zettelmeyer (2007) who painstakingly construct alternative measures of investor losses for six countries covering 246 individual debt instruments. Nonetheless, we show in the data appendix that our (much simpler) aggregate method gives very similar results when applied to the data from the countries in their sample. The relationship between defaults, settlements and output has been previously studied by Tomz and Wright (2007), who uncovered a modest tendency for defaults to occur when output was low, and for settlements to occur when output has partially returned to trend. This paper confirms those previous findings, and expands upon them by examining data on creditor losses and defaulting country debt levels, and by demonstrating that output growth in the year of default is negatively related to the length of time taken to renegotiate those debts.

Second, there has been a substantial recent theoretical literature on sovereign debt and default. Our borrowing environment, outside of debt restructuring, is a version of the Eaton and Gersovitz (1981) model of defaultable debt, which has been used recently by a large number of authors including Arellano (2007), Aguiar and Gopinath (2006) and Tomz and Wright (2007). Unlike these papers, we model the consequences of default as the endogenous outcome of a bargaining model. Other recent papers that model debt restructuring negotiations, including Yue (2006) and Bi (2007), assume that debts are extinguished upon the conclusion of negotiations and hence, unlike our theory, are unable to explain why countries typically exit default with debt levels higher than those with which they entered default. In contrast to Bi (2007) and Kovrijnykh and Szentes (2007), who's models predict that defaults should always end when output is above trend, our theory is able to match the relatively weak relationship between output and default settlements observed in the data. In contrast to Pitchford and Wright (2008), we abstract from collective action problems and focus on the role of limited commitment in producing delay.

Third, our paper contributes to the extensive literature on delays in bargaining. Like the complete information bargaining model of Merlo and Wilson (1995, 1997), which builds on the earlier work of Rubinstein (1982) and Baron and Ferejohn (1989) and which was applied to sovereign debt renegotiations by Bi (2007), our theory produces delays that are optimal from the perspective of the parties to the negotiation. Unlike these papers, our theory allows for endogenous flow payoffs during bargaining (in addition to terminal payoffs) and for outside options (the debtor can settle at any time by repaying the debt in full), and produces outcomes that may be socially suboptimal due to the presence of default risk. Unlike Shaked (1994), taking the outside option in our model is not socially wasteful, and so does not directly lead to delay (see also the discussion in Binmore, Osborne and Rubinstein 1992). Our theory features agents with different levels of patience, which Cripps (1998) has shown may produce inefficient delay in equilibrium in a stochastic production economy, in contrast to our endowment economy. Finally, we model bargaining under complete information, unlike the extensive literature on delays with bargaining under asymmetric information (surveyed in Binmore, Osborne and Rubinstein 1992), to capture our finding that restructuring outcomes are correlated with observed economic conditions.

Fourth and finally, our paper is a contribution to the literature evaluating the quantitative implications of non-cooperative bargaining games. For example, Diermeier, Eraslan and Merlo (2003) estimate a noncooperative bargaining model of European Parliamentary formation to capture the effect of political institutions on government formation and dissolution, while Eraslan (2008) estimates a bargaining model to compute the liquidation value for a firm in Chapter 11 bankruptcy. In this paper, we calibrate a bargaining model in a sovereign debt setting.

The rest of this paper is organized as follows. Section 2 describes our database of sovereign debt renegotiation outcomes, and presents our empirical findings. Section 3 outlines the environment in which sovereign borrowing and debt renegotiation take place. Section 4 takes borrowing outcomes as given and studies the debt renegotiation process, establishing existence of an equilibrium bargain and providing a sufficient condition for uniqueness. Section 5 then combines the restructuring model with the borrowing environment and provides a proof of existence of an equilibrium for the overall model. Less technical readers may prefer to skim Sections 4 and 5. Section 6 shows that a calibrated version of the model can match the facts introduced in Section 2, while Section 7 concludes and an appendix provides further detail on our database, proofs of theorems, and on our numerical algorithm for solving the model.

2. DEBT RENEGOTIATION OUTCOMES

In this section we describe our database of sovereign defaults and debt renegotiation outcomes, and present our empirical findings. Our database contains information on which countries default, how much debt they defaulted on, how these obligations were eventually settled, the timing of defaults and settlements and the economic circumstances surrounding these dates. Our database is limited in coverage by the availability of data on the size of the write-down, or haircut, taken by creditors. As a result, we begin by describing our sources for these data, before turning to a discussion of other data sources.

There is much debate but no agreed consensus for measuring the extent to which creditors experience a decline in the value of their claims. Numerous estimates of creditor write-downs have been presented in the literature, each calculated under different assumptions and each covering a small [frequently non-overlapping] set of defaults. For example, The World Bank (1993) provides estimates of "debt reduction equivalents" for nine countries which add the write-down in the face value of the debt to an estimate of the present value of interest rate reductions, and then adjust these estimates for new lending from both official and private sectors as well as the value of any new collateral. Cline (1995) criticizes the World Bank methodology for fully deducting the value of new lending, and provides estimates of debt forgiveness for nine extra countries. Many private sector groups and investment banks have also circulated estimates of creditor losses , often without clearly stating the assumptions underlying their construction (for example, the Global Committee of Argentine Bondholders, 2004, report estimates for seventeen renegotiations). The most rigorous measurement is by Sturzenegger and Zettelmeyer (2005, 2007), who provide careful instrument-by-instrument estimates of creditor losses for 246 debts involved in a set of six renegotiations.

The small number of estimates of creditor losses, combined with the fact they have been calculated under different assumptions, led us to construct our own series of estimates on a common methodological basis. The broadest source of data is the World Bank's *Global Development Finance* (GDF) publication, which, starting in 1989, reports estimates of debt stock reduction, and interest and principal forgiven, as well as debt buybacks, and which we combine to form an estimate of debt forgiveness following the methodology in Cline (1995). The standardization of this data across very different renegotiations and its broad coverage make it the natural starting point for a broad analysis of renegotiation outcomes. Moreover, as we show in the Appendix, the estimates we calculate correlate closely with those presented in the other studies listed above.

The World Bank data do not make any distinction between forgiveness of debts by private creditors and forgiveness by official creditors. However, both our theory of default and renegotiation, and some of our other data, correspond most directly to debt owed to private creditors. To control for this, we obtained data on the total amount of debt renegotiated, as well as that proportion owed to private creditors from both GDF and IIF (2001) and used this ratio to scale down our estimates of debt forgiveness. The calculated level of debt forgiveness was then expressed as a proportion of the stock of debt owed to private creditors that participated in renegotiations.

The resulting series on private creditor haircuts covers ninety defaults and renegotiations by seventy-three separate countries that were completed after GDF data on debt forgiveness first became available in 1989 and that ended prior to 2006. Default start and end dates were taken from the series published by Standard and Poors (described in Beers and Chambers 2006) and combined defaults on both bank debts and bonds. The Standard and Poors data record only the year in which a default started and ended, and so we supplement these dates with data from Arteta and Hale (2007) and Trebesch (2008), as well as a range of primary sources, to come up with the month, and in some cases the day, in which a default started and ended. The entire list of defaults and haircuts is tabulated in the Data Appendix. These data were combined with annual data on public and publicly guaranteed long term debt, taken from GDF, as well as various indicators of economic activity taken from the World Bank's *World Development Indicators* publication.

Table 1 presents some summary statistics on the length of time taken to settle a default, which we refer to as delay, and on average haircuts weighted by the level of outstanding debt. The measurement of delay is complicated by the possibility that a country may settle a previous default and immediately default again. Such consecutive defaults occur three times in our sample, and if we treat such instances as distinct defaults, there are ninety defaults in our sample lasting an average of 7.5 years. Delays rise to an average of 8.1 years if consecutive defaults are combined into a single default event. This finding is consistent with those of other authors, such as Pitchford and Wright (2008) who find that defaults took an average of 8.8 years to settle over the entire period from the end of the Napoleonic Wars to the present, with the average declining to 6.5 years when attention is restricted to the period after 1976 when the Foreign Sovereign Immunity Act was passed. This leads to our first result:

Fact 1: *sovereign defaults are time consuming to resolve*, taking more than eight years on average in our sample.

Table 1 also presents evidence on the average size of haircuts, where the average is weighted by the value of outstanding debts. As shown in the Table, the average creditor group experienced a haircut of 44 per-cent of the value of the debt. Further information on the sizes of haircuts and delays is presented in Figure 2 which contains a scatter plot of haircuts and delays for each of the ninety settlements contained in our sample. As shown in the Figure, haircuts in our sample have ranged from approximately zero all the way up to ninety per-cent of the value of creditors claims in the case of some African defaults. Likewise, there is a great deal of variation in delays with many defaults being settled almost immediately while others are settled in excess of two decades. Even though the mean is eight years, the mode of this data is less than one year. There is also a noticeable positive relationship between the amount of delay in renegotiation and the size of the haircut, with the correlation coefficient between the two series equalling 0.66. This gives rise to our next two results:

Fact 2: *creditor losses (or* haircuts*) are substantial*, with the average creditor experiencing a reduction in the value of their claim of forty-four per-cent.

Fact 3: *longer defaults are associated with larger haircuts*, with a correlation between the length of the renegotiation process and the size of the creditor haircut of two-thirds.

Facts 2 and 3 suggest that there may be a common factor driving both longer defaults and larger haircuts. Table 1 also presents evidence on the relationship between delays and haircuts and the level of economic activity in the year of the default. In particular, the third column shows that the larger is the decline in output in the year of default, the longer the delay and the larger the haircut, on average. The relationship is only modest, however, never rising above 0.3 in absolute value, with the correlation to haircuts bare different from zero. The fourth column Table 3 presents the relationship between delays and haircuts and the growth of output in the two years surrounding the default and finds a stronger negative relationship with haircuts. This leads to our fourth fact:

Fact 4: *larger output declines in the year of default are associated with modestly longer defaults and larger haircuts*, with correlation coefficients around -0.20

Table 3 provides further evidence on the relationship between defaults, settlements and output. As shown in the first column, there is a broad tendency for default to be associated with adverse economic conditions, with a mean level of output roughly one-half of one per-cent below trend[1], while output in non-default periods is above trend by an equal amount on average. Economic adversity is particulary likely in the first year of a default, when output was on average 1.3 per-cent below trend, and tends to have dissipated by the time a country settles with its creditors when output is on average only 0.2% below trend. Nonetheless, there is a great deal of variation across country experiences so that the overall relationship between output and default is quite weak. In almost one-third of cases, a country defaults with output above trend. This confirms the earlier finding of Tomz and Wright (2007) for a larger sample of defaults, and leads to our fifth result:

[1] Calculated using a Hodrick-Prescott filter with smoothing parameter 6.25 for annual data; see Ravn and Uhlig 2002

Fact 5: *defaults are somewhat more likely to occur when output is below trend, and settlements tend to occur when output has returned to trend,* with 64% of defaults beginning when output is below trend, and 49% ending when output is above trend. The average deviation of output from trend is -1.3% in the first year of a default, and -0.2% in the year of the settlement.

Finally, Table 3 also explores the relationship between defaults and debt levels for the defaulting country. As shown in the table, being in default is associated with levels of debt to GDP that are more than seventy per-cent higher than for when a country is not in default, bearing in mind that our sample of countries is conditioned upon having defaulted once during this period. Strikingly, the table reveals that countries tend to exit default with levels of debt that are thirty-nine percent higher than they possessed when they entered default. From this we conclude that renegotiations are ineffective at reducing the indebtedness of a debtor country. This leads to our sixth and final result:

Fact 6: *default resolution is associated with increased country indebtedness*, with the average country exiting default with a debt to GDP ratio forty per-cent higher than before they entered default.

Finally, Table 1 also shows that delays and haircuts are essentially unrelated to the initial level of indebtedness of a country. In our theory, which we begin to outline in the next section, we therefore do not focus upon differences in debt levels as a factor in negotiations.

3. ENVIRONMENT

In this section, we present our theory of sovereign borrowing and default. In our theory, as in the bulk of the literature that has followed the work of Eaton and

Gersovitz (1983), a sovereign country borrows against its future income, and in order to smooth its consumption. International credit markets are competitive, and offer one period loans that specify a state-non-contingent return as long as the contract is honored. Defaults may occur whenever it is in the best interests of the country. In our model, this may reflect either a deterioration in economic conditions (as in the traditional literature) or an improved bargaining position for the country.

In contrast to the existing literature, when a country defaults it must bargain with its creditors before it can reaccess international credit markets. A settlement between the defaulting country and its creditors may involve the issue of new debt. Delay in settlements can arise as either creditors or debtors wait for an improvement in the bargaining environment which may result from an improvement in economic conditions, or a change in bargaining power. The challenge for the model is to simultaneously reproduce the observed delay in bargaining, the observed distribution of settlement terms, and the observed relationship between these bargaining outcomes and economic conditions.

We begin by first describing the decisions facing a sovereign country that is in good standing with its creditors, before moving on to a description of international credit markets. Since the formulation of the model for a debtor that is not in default is relatively standard, we do not present it in great detail. We then turn to a detailed description of the bargaining environment.

3.1 The Sovereign Borrower

Consider a world in which time is discrete and lasts forever. In each period $t = 0, 1, ..,$ a sovereign country receives an endowment of the single non-storeable consumption good $e(s)$ that is a function of the exogenous state s which takes on values in the finite set S. Thus, the endowment also takes on only a finite number,

N_e, of values. The state s summarizes all sources of uncertainty in the model and evolves according to a first order Markov process with transition probabilities given by a transition matrix with representative element $\pi(s'|s)$. Specifically, the evolution of the state s governs both the evolution of the country's endowment, as well as the evolution of the country's bargaining position with creditors, which are the only uncertain objects in the model.

The sovereign country is represented by an agent that maximizes the discounted expected value of its utility from consuming state contingent sequences of the single tradeable consumption good $\{c_t(s^t)\}$ according to

$$E\sum_{t=0}^{\infty}\beta^t\sum_{s^t|s_0}\pi\left(s^t|s_0\right)U\left(c_t\left(s^t\right)\right).$$

Here, the felicity function U is twice continuously differentiable, strictly increasing and strictly concave so that the country is averse to fluctuations in its consumption. We allow for the possibility that U is defined over negative levels of c, reflecting the idea that it is possible to export more than the country's endowment of the tradeable good by using a (potentially very costly) technology for converting otherwise non-tradeable goods into tradeables goods. The discount factor β lies between zero and one and is assumed to imply a discount rate in excess of the world interest rate. As a result, international borrowing may be motivated by both a desire to smooth consumption, as well as a desire to tilt a country's consumption profile forward in time.

As long as the sovereign country is not in default, we will say that the country is in good standing with its international creditors. A country in good standing with it's creditors enters the period with new value of the state s, and a level of international debt b which is constrained to lie in the finite set B which has cardinality N_b. It is assumed that the set of debt levels, B, contains both negative and positive elements, as well as the zero element, where negative elements are interpreted as savings by the country. The sovereign's first decision is whether or not to default on its debts. If the sovereign borrower defaults, they receive a payoff given by $\tilde{V}^D(b,s)$, which is a N_e by N_b vector of real numbers, and which will be determined below when we describe the process by which a country in default bargains with its creditors.

Let $V(b,s)$ denote the value function of a country of that enters the period with debt b and state s, *before* the country has decided whether or not to default, which is an N_e by N_b vector of real numbers. Similarly, let $V^R(b,s)$ denote the value function of a country that enters the period with debt b and state s, *after* it has decided to repay it's debts, which is also an N_e by N_b vector of real numbers. Then the value function $V(b,s)$ obeys the relationship

$$V(b,s) = \max\left\{V^R(b,s), \tilde{V}^D(b,s)\right\}. \tag{1}$$

If the sovereign country repays its debts, it must then decide how much to consume c and how much debt $b' \in B$ to take into the next period. The value function associated with the repayment of debt, V^R, is an $N_e \times N_b$ vector which obeys the recursive relationship

$$V^R(b,s) = \max_{c,b'\in B} U(c) + \beta \sum_{s'\in S} \pi(s'|s) V(b',s'), \tag{2}$$

subject to

$$c - q(b',s)b' \leq e(s) + b.$$

Here, $q(b',s)$ is a $N_e \times N_b$ vector of prices today of a bond that pays one unit tomorrow as long as the country does not default, and that depends on the current state s and total borrowing b'. It is determined by competition in international credit markets, which we describe in the next subsection.

3.2 International Credit Markets

We assume that international credit markets are populated by a large number of risk neutral creditors that behave competitively. The opportunity cost of funds for a creditor is given by the world interest rate r^w, which we assume is constant. Competition in the international credit market ensures that creditors expect to earn the world interest rate from their investments in the sovereign borrower's bonds.

To understand the determinants of the price of a country's bonds, suppose the country issues a total of b claims, each of which pays one unit tomorrow as long as the country does not default. If a creditor were to buy one unit of the country's bonds at price $q(b, s)$, then competition ensures that they must expect to receive $(1 + r^w) q(b, s)$ on average tomorrow. The actual return they receive has two components. First, with some probability $1 - p(b, s)$ the country is expected to repay-in-full tomorrow which yields a total of one unit. Second, with probability $p(b, s)$ the country defaults. In this case, the country will commence bargaining with its creditors and the creditor will receive a one-in-b share of any returns from this bargaining process. If we let $\tilde{W}(b, s')$ be a $N_e \times N_b$ vector of the total expected discounted values of any settlement on a default on b bonds in state s' tomorrow, viewed from the perspective of tomorrow, then the equilibrium bond price must satisfy

$$q(b, s) = \frac{1 - p(b, s) + p(b, s) \sum_{s' \in S} \pi(s'|s) \tilde{W}(b, s')/b}{1 + r^w}.$$

The total expected discounted value of any settlement, viewed from tomorrow, $\tilde{W}(b, s')$ will be determined along with the $N_e \times N_b$ vector of values to the country from default $\tilde{V}^D(b, s)$, as a result of the bargaining process which we describe in the next section. For now, we assume that $\tilde{W}(b, s')$ is bounded below by zero and above by b, which in turn ensures that the bond price function takes values in the interval $[0, 1/(1 + r^w)]$. We let $\mathcal{Q}(B \times S)$ be the set of all functions on $B \times S$ taking values in $[0, 1/(1+r^w)]$.

It remains to describe the probability of default $p(b,s)$, which is determined by the sovereign's decision to default described in (1) above. For most values of (b,s), the sovereign country will strictly prefer defaulting over repaying, or repaying over defaulting. However, it is possible that for some values of (b,s) that the country is indifferent. To deal with this possibility, we define an indicator correspondence for default with debt b in state s, $\Phi(b,s)$, as

$$\Phi(b,s) = \begin{cases} 1 & \text{if } \tilde{V}^D(b,s) > V^R(b,s) \\ 0 & \text{if } \tilde{V}^D(b,s) < V^R(b,s) \\ [0,1] & \text{if } \tilde{V}^D(b,s) = V^R(b,s) \end{cases}.$$

From this we can define the default probability correspondence for debt b and state s, $P(b,s)$, as the set of all $p(b,s)$ constructed as

$$p(b,s) = \sum_{s' \in S} \phi(b,s') \pi(s'|s),$$

for some $\phi(b,s) \in \Phi(b,s)$.

3.3 Restructuring Debt through Negotiations

In this subsection, we specify the process by which a sovereign country in default bargains with its creditors over a settlement. We abstract from the coordination problems in debt restructuring negotiations studied by Pitchford and Wright (2008) and others, and assume that creditors are able to perfectly coordinate in bargaining with the country. Hence, our restructuring negotiations are modeled as a game between two players: the sovereign borrower in default, and a single creditor.

We assume that the country is in autarky in the period in which the default actually occurs. Hence the relationship between the total value to creditors from a settlement

$\tilde{W}(b,s')$ and the value to the country from default $\tilde{V}^D(b,s)$, that we introduced above, and the $N_e \times N_b$ vectors of outcomes of bargaining that we derive below, $W(b,s')$ and $V^D(b,s)$, is given by

$$\begin{aligned}
\tilde{W}(b,s) &= \delta E\left[W(b,s')\,|s|\right], \\
\tilde{V}^D(b,s) &= U\left(e^{def}(s)\right) + \beta E\left[V^D(b,s')\,|s\right].
\end{aligned}$$

Here, $\delta = \frac{1}{1+r^w}$ and $e^{def}(s)$ is used to denote the possibility that the endowment process may be different in the event of a default (often it is assumed to be lower than when not in default, reflecting an assumed direct cost of default).

The timeline of actions is described in Figure 4. Negotiations begin with a sovereign country that has previously entered default with a level of debt b. At stake is the ability of the country to reaccess credit markets. The value to the country of settling today in state s with its creditors and re-accessing capital markets with a new level of debt b' is given by $\sum_{s'\in S}\pi(s'|s)\,V(b',s')$, where V was described above and is treated as exogenous for the purposes of bargaining. This value represents the future surplus associated with reaching an agreement.

Neither player is able to commit to a split of surplus beyond the current period. Instead, the players can only agree to a current transfer of resources that may be partially (or wholly) financed by the issue of new debt securities. The ability to share future surplus is therefore limited by the fact that the country may default on these new debt securities in the future. Delay can occur as both the creditor and the debtor wait for an improvement in the terms under which new debt securities can be issued. Importantly, the same commitment problems that lead to default also drives the outcome of the renegotiation.

If delay occurs, the bargaining game continues with a new state s' and the same level of debt b tomorrow. The assumption that the amount of debt in default, b, is unchanged throughout negotiations captures the fact that for most of the period under study, interest on missed payments was not a part of default settlements[2].

Negotiations between the creditors and the debtor are efficient, in the sense that agreements are optimal for the two parties subject to the constraints on negotiations implied by future default risk. To capture this fact, we say that negotiations are *privately optimal ex post*. Nonetheless, delay may be said to be *socially wasteful ex post*, as the country is unable to access capital markets while in default, and thus forfeits potential gains from trade in tilting and smoothing it's consumption. Delay may also be *socially and privately beneficial ex ante*, as a costly default ex post may lead to more socially desirable borrowing outcomes ex ante. We return to this issue below.

3.3.1 Timing and Strategies.—

Bargaining occurs according to a randomly alternating offer bargaining game with an outside option available to the debtor, with timing illustrated in Figure 5. At any point, the debtor country has the option of paying off the defaulted debt in full, using any desired mix of current transfers and new debt securities issued at the market price. We refer to this action as the *outside option* of the debtor, although we stress that this is strictly only an outside option for the game conditional on default, and not for the entire borrowing environment. In addition to being a feature of the actual environment governing sovereign debt renegotiations, this assumption guarantees that the total value of the settlement never exceeds b which serves to bound our bond price function.

In every period and in each state of the world s, either the sovereign borrower or the creditor is selected to be the *proposer* who then proposes a settlement offer. A *proposal* consists of a settlement with creditors that is composed of a transfer of

[2] In cases that went to court, the courts did not award interest on missed payments until 1997 as part of the legal proceedings involving Elliot Associated and Peru.

resources τ to the creditor in the current period, and an issue of new debt securities b'. The proposer's action is therefore given by an offer of two values $(\tau, b') \in \mathbb{R} \times B$. We do not place any additional bounds on the issue of new debt, although debt issues will continue to be limited by the price that new creditors will be prepared to offer for these new bonds. Importantly, we allow for the possibility that the settlement may contain an amount of "new money" in which the country receives a positive flow of the consumption good in the period in which they settle (this corresponds to a negative τ).

Once a proposal is made, the non-proposing agent chooses to either *accept* or *reject* the current proposal. If the proposal is accepted, or if the debtor country's outside option is taken, the bargaining concludes and the country emerges from default with the new negotiated debt level. If the proposal is rejected and the outside option is not taken, the game continues to the next period, and we say that there has been *delay in bargaining*. In the next period, the proposer is chosen following the realization of the next period's state variable and the value of the debt is unchanged. The timing then repeats with the next proposer suggesting an offer. In Figure 5, we depict the timeline of the bargaining game.

A history of the bargaining game is a list of all previous actions and states that have occurred after a country's most recent default. If a non-proposer accepts the most recent offer or if the outside option is chosen, the negotiation ends and there is no reason to track the history. If no offer has been accepted, and if t indexes stages, a history up to the beginning of stage t is defined by the sequence of realizations for the state variable and the sequence of rejected offers:

$$h^t = \left\{ s^t = (s_0, s_1, ..., s_{t-1}), (\tau, b')^t = \left((\tau_0, b'_0), (\tau_1, b'_1), ..., (\tau_{t-1}, b'_{t-1})\right) \right\}.$$

We let H^t denotes the set of all histories to stage t.

Strategies map the level of the defaulted debt b and the history into a choice of actions. The current state determines the identity of the current proposer, and the set of feasible actions depends on which player is the proposer. A strategy for the creditor when they are the proposer is a function $\sigma^{C,P} : B \times H^t \times S \rightarrow \mathbb{R} \times B$. The situation is more complicated when the debtor is the proposer due to the fact that the debtor may elect to take the outside option. In particular, a strategy for the debtor when they are the proposer is a function $\sigma^{D,P} : B \times H^t \times S \rightarrow \mathbb{R} \times B \times \{0, 1\}$, where the third element takes on the value one if the debtor takes the outside option; whether or not the debtor takes the outside option, there is an associated transfer and new debt level (τ, b'). A strategy for creditor when they are not the proposer depends on whether or not the debtor has taken the outside option. If the debtor has *not* taken the outside option, a strategy for a non-proposing creditor is a function $\sigma^{C,NP} : B \times H^{t+1} \rightarrow \{0, 1\}$ where 0 denotes rejection of the proposal, and 1 acceptance of the proposal. If the debtor has taken the outside option, the creditor has no choice but to accept the proposed settlement and so a strategy for a non-proposing creditor is a function $\sigma^{C,NP} : B \times H^{t+1} \rightarrow \{0\}$. A strategy for the debtor when they are not the proposer is a function $\sigma^{D,NP} : B \times H^{t+1} \rightarrow \{0\} \cup \{1\} \cup \{2\} \times \{(\tau, b') \in \mathbb{R} \times B : \tau + q(b', s_{t+1}) b' \geq b\}$ where 0 indicates a rejection, 1 indicates acceptance, and 2 indicates that the outside option was chosen with associated transfer and new debt levels (τ, b'). A strategy profile is a pair of strategies, one for each player.

Next we discuss outcomes and payoffs. An outcome for us is a termination of negotiations plus the final accepted offer. That is, an *outcome* is a stopping time t^* and the associated proposal (τ, b'). At any history, a strategy profile induces an outcome and hence a payoff for each player. The payoff to the debtor given outcome $\varphi = \{t^*, (\tau, b')\}$ after history s^{t^*} is

$$V^D\left(t^*, s^{t^*}, (\tau, b')\right) = \sum_{r=0}^{t^*-1} \beta^r U\left(e^{def}(s_r)\right) + \beta^{t^*}\left\{U\left(e^{def}(s_{t^*}) - \tau\right) + \beta E\left[V(b', s_{t^*+1}) | s_{t^*}\right]\right\},$$

while to the creditor it is given by

$$W\left(t^*, s^{t^*}, (\tau, b')\right) = \delta^{t^*}\left\{\tau + q\left(b', s_{t^*}\right) b'\right\}.$$

Let $G(h^t)$ denote the game from date t onwards starting from history h^t. Let $|h^t$ denote the restriction to the histories consistent with h^t. Then $\sigma|h^t$ is a strategy profile on $G(h^t)$. We let $\varphi\left(\sigma|h^t\right)$ be the outcome generated by the strategy profile $\sigma|h^t$ in game $G\left(h^t\right)$. A strategy profile is subgame perfect (SP) if, for every history h^t, $\sigma|h^t$ is a Nash equilibrium of $G(h^t)$. That is

$$\begin{aligned} W\left(\varphi\left(\sigma|h^t\right)\right) &\geq W\left(\varphi\left(\sigma^D|h^t, \sigma^{C\prime}|h^t\right)\right), \\ V^D\left(\varphi\left(\sigma|h^t\right)\right) &\geq V^D\left(\varphi\left(\sigma^{D\prime}|h^t, \sigma^C|h^t\right)\right), \end{aligned}$$

for all σ, t, and h^t.

As is customary in the literature, we impose the restriction of stationarity. A strategy profile is stationary if the actions prescribed at any history depend only on the current state and proposal. That is a *stationary strategy profile* satisfies:

$$\begin{aligned} \sigma^D\left(h^t, s_t\right) &= \sigma^D\left(s_t\right) \\ \sigma^C\left(\left(h^t, \left(s_t, \left(\tau_t, b_t'\right)\right)\right)\right) &= \sigma^C\left(s_t, \left(\tau_t, b_t'\right)\right), \end{aligned}$$

for all h^t and all t. A *stationary subgame perfect equilibrium* (SSP) outcome and payoff are the outcome and payoff generated by an SSP strategy profile. We define a stationary outcome as $\left((B \times S)^\mu, \mu\right)$,where $\mu = (\tau, b')$ and where $(B \times S)^\mu$ is the set of debt levels b and states s on which an agreement occurs or the outside option is taken, and where $(B \times S) \setminus (B \times S)^\mu$ is the disagreement set.

4. SOLUTION TO THE BARGAINING MODEL

The solution to the overall model involves solving a fixed point problem. First, taking as given the solution to the bargaining problem, we solve for the solution to the debtor countries default problem and update the market price of debt. Second, we take the market price of debt and the debtor's value function from repayment and then use these to solve the bargaining problem. A solution is a fixed point to a composition of the associated operators.

In this section, we focus on the bargaining model, taking as given the form of the solution to the borrowing problem. After establishing a recursive representation for the solution of this model, we then establish the existence of a solution and give some sufficient conditions for uniqueness of the equilibrium bargain. We also provide an initial characterization of the SSP outcome. In the next section, we return to the borrowing model and establish the existence of a solution to the entire model. We then illustrate possible equilibrium dynamics through a series of examples designed to show features of the model which produce delay in bargaining.

4.1 Recursive Problem Statement

For this section, we take the solution to the borrowing problem as given. That is, the debtor country's value of accessing capital markets $V(b,s)$ is assumed to be a fixed element of the set all real valued N_e by N_b vectors, and the equilibrium bond price function $q(b,s)$ is assumed to be a fixed element of $\mathcal{Q}(B \times S)$. Given these assumptions, we then show that the SSP values of the bargaining game are fixed points of a particular functional equation. As is usual, the key to the approach is that we focus directly on the outcomes of the SSP, rather than on the SSP itself.

In particular, take the SSP outcome, which consists of a set of states in which acceptance occurs and the proposal that is accepted in that state, $((B \times S)^{\mu}, \mu)$, as given. We can define the value of this outcome as follows. First, fix the value of the defaulted debt to b. Then, given a sequence of realizations of the state, define the stopping time for an agreement by t^* where $(b, s_{t^*}) \in (B \times S)^{\mu}$ and $(b, s_t) \in$

$(B \times S) \setminus (B \times S)^{\mu}$ for all $t = 0, ..., t^* - 1$. Then we can define the value of this outcome in state s as

$$
\begin{aligned}
& v^{\mu}(b, s) \\
\equiv & \begin{pmatrix} v_1^{\mu}(b, s) \\ v_2^{\mu}(b, s) \end{pmatrix} \\
= & \begin{pmatrix} E\left[\sum_{t=0}^{t^*-1} \beta^t U\left(e^{def}(s_t)\right) + \beta^{t^*}\left\{U\left(e^{def}(s_{t^*}) - \tau(s_{t^*})\right) + \beta V\left(b(s_{t^*}), s_{t^*}\right)\right\} |s\right] \\ E\left[\delta^{t^*}\left\{\tau(s_{t^*}) + b(s_{t^*}) q\left(b(s_{t^*}), s_{t^*}\right)\right\} |s\right] \end{pmatrix}.
\end{aligned}
$$

These outcomes are our V^D and W functions defined above.

First, we establish that the value function $v^{\mu}(b, s)$ is the unique function defined on $B \times S$ taking values in $\mathbb{R}^2$ satisfying a particular functional equation. The proof relies on the following mapping which is defined for an arbitrary stationary outcome. Specifically, consider the mapping T on the set of functions $f : B \times S \to \mathbb{R}^2$ into itself defined by:

$$
Tf_1(b, s) = \left\{ \begin{array}{cc} u(e^{def}(s) - \tau(b, s)) + \beta E[V(b'(b, s), s'|s)] & \text{if } (b, s) \in (B \times S)^{\mu} \\ u(e^{def}(s)) + \beta E[f_1(b, s')|s] & \text{if } (b, s) \in (B \times S) \setminus (B \times S)^{\mu} \end{array} \right\},
$$

and

$$
Tf_2(b, s) = \left\{ \begin{array}{cc} \tau(b, s) + b'(b, s) q(b'(b, s), s) & \text{if } (b, s) \in (B \times S)^{\mu} \\ \delta E[f_2(b, s')|s] & \text{if } (b, s) \in (B \times S) \setminus (B \times S)^{\mu} \end{array} \right. .
$$

The first operator applies to the payoff of the debtor country, and simply states that if (b, s) is in the set $(B \times S)^{\mu}$, which is the set of debt levels and states in which either the outside option is taken or a proposal is accepted, then the payoff to the country is found by evaluating the value of that proposal. Conversely, if (b, s) is not in the acceptance set, the debtor country consumes its endowment in default today and the discounted value of the expected payoff from continuing the bargaining game tomorrow. The second operator is similar and applies to the payoff of the creditors.

Lemma 1 *Given an outcome* $((B\times S)^{\mu},\mu)$ *where* $\mu=(\tau,b)$, v^{μ} *is the unique function defined on* $B\times S$ *taking values in* $\mathbb{R}^2$ *for which*

$$\begin{pmatrix} v_1^{\mu}(b,s) \\ v_2^{\mu}(b,s) \end{pmatrix} = \begin{cases} \begin{pmatrix} u(e^{def}(s)-\tau(b,s))+\beta E[V(b'(b,s),s')|s] \\ \tau(b,s)+b'(b,s)\,q(b'(b,s),s) \qquad \text{if } s\in S^{\mu} \end{pmatrix} \\ \begin{pmatrix} u(e^{def}(s))+\beta E[v_1^{\mu}(b,s')|s] \\ \delta E[v_2^{\mu}(b,s')|s] \qquad \text{if } s\in S\backslash S^{\mu} \end{pmatrix} \end{cases}.$$

Proof. The proof requires us to show that v^{μ} is a fixed point of the operator T, and that the operator T has a unique fixed point. First, to see that v^{μ} is a fixed point, note that if $(b,s_0)\in(B\times S)^{\mu}$ then

$$Tv^{\mu}(b,s_0)=\begin{pmatrix} u(e^{def}(s_0)-\tau(b,s_0))+\beta E[V(b'(b,s_0),s_1|s_0)] \\ \tau(b,s_0)+b'(b,s_0)\,q(b'(b,s_0),s_0) \end{pmatrix},$$

which is precisely the definition of v^{μ} on states for realizations in which the stopping time is zero. Alternatively, suppose that $(b,s_0)\in(B\times S)\setminus(B\times S)^{\mu}$. Then by definition of T we have

$$T\begin{pmatrix} v_1^{\mu}(b,s_0) \\ v_2^{\mu}(b,s_0) \end{pmatrix} = \begin{pmatrix} u(e^{def}(s_0))+\beta E[v_1^{\mu}(b,s_1)|s_0] \\ \delta E[v_2^{\mu}(b,s_1)|s_0] \end{pmatrix}.$$

Define a stopping time t^*,such that if (b,s_0) is the initial state, t^* is the period in which agreement is reached. That is, $(b,s_{t^*})\in(B\times S)^{\mu}$ and $(b,s_t)\in(B\times S)\setminus(B\times S)^{\mu}$ for all $t<t^*$. Then iterating on the operator T we have

$$\begin{aligned} & T\begin{pmatrix} v_1^{\mu}(b,s_0) \\ v_2^{\mu}(b,s_0) \end{pmatrix} \\ = & \begin{pmatrix} u(e^{def}(s_0))+\beta E[v_1^{\mu}(b,s_1)|s_0] \\ \delta E[v_2^{\mu}(b,s_1)|s_0] \end{pmatrix} \\ = & \begin{pmatrix} u(e^{def}(s_0))+\beta E[\sum_{t=1}^{t^*-1}\beta^t u\left(e^{def}(s_t)\right)+\beta^{t^*}E[v_1^{\mu}(b,s_{t^*})\,|s_1]|s_0] \\ \delta\left[E\left[E[\delta^{t^*}v_2^{\mu}(b,s_{t^*})|s_1]\right]|s_0\right] \end{pmatrix} \end{aligned}$$

$$= \begin{pmatrix} E\left[\sum_{t=0}^{t^*-1} \beta^t U\left(e^{def}\left(s_t\right)\right) + \beta^{t^*}\left\{U\left(e^{def}\left(s_{t^*}\right) - \tau\left(b, s_{t^*}\right)\right) V\left(b'\left(b, s_{t^*}\right), s_{t^*}\right)\right\} |s\right] \\ E\left[\delta^{t^*}\left\{\tau\left(b, s_{t^*}\right) + b'\left(b, s_{t^*}\right) q\left(b'\left(b, s_{t^*}\right), s_{t^*}\right)\right\} |s\right] \end{pmatrix} = v^\mu.$$

Second, to show that T has a unique fixed point it is sufficient to show that T is a contraction on the metric space of functions defined on $B \times S$ taking values in $\mathbb{R}^2$ endowed with the sup (or in this case, the max) norm. That is, we require that if f^1 and f^2 are each functions mapping $B \times S$ into $\mathbb{R}^2$, then

$$||T(f^1) - T(f^2)||_\infty \leq \delta||f^1 - f^2||_\infty.$$

To see this, note that if $(b, s) \in (B \times S)^\mu$, then Tf is independent of the function f and hence

$$\left|Tf^1(b,s) - Tf^2(b,s)\right| = \max\left\{\left|Tf_1^1(b,s) - Tf_1^2(b,s)\right|, \left|Tf_2^1(b,s) - Tf_2^2(b,s)\right|\right\} = 0.$$

Otherwise,

$$\begin{aligned} & \left|Tf^1(b,s) - Tf^2(b,s)\right| \\ = \ & \max\left\{\left|\beta E[f_1^1(b,s')|s] - \beta E[f_1^2(b,s')|s]\right|, \left|\delta E[f_2^1(b,s')|s] - \delta E[f_2^2(b,s')|s]\right|\right\} \\ = \ & \beta \max\left\{\left|E[f_1^1(s') - f_1^2(s')|s]\right|, \left|E[f_2^1(s') - f_2^2(s')|s]\right|\right\} \\ \leq \ & \beta||f^1 - f^2||_\infty, \end{aligned}$$

where we have exploited our assumption that $\beta < \delta < 1$. But then

$$\left\|Tf^1 - Tf^2\right\|_\infty = \max_{b,s}\left|Tf^1(b,s) - Tf^2(b,s)\right| \leq \beta||f^1 - f^2||_\infty.$$

■

The result of Lemma 1 is helpful in establishing the properties of an second operator that we will use to characterize equilibria. Given any pair of functions (f_1, f_2) with $f_i : B \times S \rightarrow \mathbb{R}$ for $i = 1, 2$, we define the mapping $\hat{T}$ as follows: If s is such that the debtor is the proposer

$$\begin{aligned}
&\hat{T}f_1(b,s) \\
&= \max\left\{ \begin{array}{c} \max_{\tau,b'} u(e^{def}(s)-\tau)+\beta E[V(b',s')|s] \\ s.t \quad \tau+b'q(b',s)\geq \min\{b,\delta E[f_2(b,s')|s]\} \end{array}, u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right] \right\},
\end{aligned}$$

and

$$\hat{T}f_2(b,s)=\min\{b,\delta E[f_2(b,s')|s]\}$$

while if s is such that the creditor is the proposer

$$\begin{aligned}
&\hat{T}f_2(b,s) \\
&= \max\left\{ \min\left\{ b, \begin{array}{c} \max_{\tau,b'} \tau+b'q(b',s) \\ s.t \quad u(e^{def}(s)-\tau)+\beta E[V(b',s')|s] \\ \geq u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right] \end{array} \right\}, \delta E\left[f_2(b,s')|s\right] \right\},
\end{aligned}$$

and

$$\begin{aligned}
&\hat{T}f_1(b,s) \\
&= \max\left\{ u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right], \begin{array}{c} \max_{\tau,b'} u(e^{def}(s)-\tau)+\beta E[V(b',s')|s] \\ s.t \quad \tau+b'q(b',s)\geq b \end{array} \right\}
\end{aligned}$$

Intuitively, the $\hat{T}$ mapping yields the values from bargaining at a given stage with defaulted debt b and current state s, given that the continuation values associated with not reaching agreement this period are determined by f_1, for the debtor, and f_2 for the creditor. To understand this mapping, note that if the debtor is the proposer, they have three options. First, they could make an offer which will not be accepted. In this case, the debtor consumes the autarky endowment level this period and moves on the next stage with defaulted debt still at b, new state s' and payoffs encoded in f_1, while the creditor receives nothing today and a future payoff encoded by f_2. This payoff is the right hand component of the debtor-proposer half of the operator, for both the debtor and the creditor.

Second, the debtor could take the outside option, in which case the creditor receives the value of the defaulted debt b, and the debtor recives the maximum value acheivable while still delivering a payoff of b to the creditor. This corresponds to the left hand side of the creditors part of the debtor-proposer half of the operator, and to the left hand side of the debtor's part of the operator given the constraint on creditor utility defined by b.

Third, the debtor could make an offer that is accepted. In this case, since the debtor makes the offer, the creditor receives none of the surplus from the agreement, and hence receives the same payoff as if the offer was not accepted (the right hand side of the creditor part of the debtor-proposer half of the operator). The debtor, on the other hand, receives the maximum value that can be acheived while delivering this value to the creditor (the left hand side of the debtor's part of the operator with the constraint defined by the reservation payoff of the creditor). Since the debtor would never take the outside option when it can do better by making an offer that is accepted, the minimum over the value of the debt and the creditors reservation value is the relevant determinant of the constraint.

Similar logic underlies the half of the operator that applies to states in which the creditor is the proposer, noting that the debtor will extract all of the surplus from an accepted proposal up to a maximum value of b at which level the debtor will take the outside option.

The following theorem establishes an equivalence between SSP payoffs and fixed points of the $\hat{T}$ operator.

Theorem 2 *The functions $f = (f_1, f_2)$ are SSP payoffs if and only if $\hat{T}f = f$.*

Proof. First, suppose that f are SSP payoffs. Fix $(b, s) \in B \times S$. Suppose that no proposal is accepted at (b, s), and the outside option is not taken. Then the SSP payoffs f satisfy the relationships

$$\begin{aligned} f_1(b,s) &= u(e^{def}(s)) + \beta E\left[f_1(b,s')|s\right], \\ f_2(b,s) &= \delta E\left[f_2(b,s')|s\right]. \end{aligned}$$

If a proposal is accepted at (b,s), it must be that it gives the agent who receives the proposal at least their reservation utility. If the debtor is proposing, then it must be that the proposal (τ, b') satisfies

$$\tau + b'q(b',s) \geq \min\left\{b, \delta E[f_2(b,s')|s]\right\} = \delta E[f_2(b,s')|s],$$

while if the creditor is proposing, it must satisfy

$$\begin{aligned} & u(e^{def} - \tau) + \beta E[V(b',s')|s] \\ \geq \; & \max\left\{ u(e^{def}(s)) + \beta E\left[f_1(b,s')|s\right], \; \begin{array}{c} \max_{\tau,b'} u(e^{def}(s) - \tau) + \beta E[V(b',s')|s] \\ s.t \;\; \tau + b'q(b',s) \geq b \end{array} \right\}. \end{aligned}$$

Moreover, as the proposal is part of a SSP, it must give the proposer the largest payoff over all such feasible proposals. Hence, if the debtor proposes in a state where a proposal is accepted

$$\begin{aligned} f_1(b,s) &= \max_{\tau,b'} u(e^{def}(s) - \tau) + \beta E[V(b,s')|s], \\ & \quad s.t. \; \tau + b'q(b',s) \geq \min\left\{b, \delta E[f_2(b,s')|s]\right\}, \end{aligned}$$

while if a creditor proposes, it must be that

$$f_2(b,s) = \min\left\{ b, \; \begin{array}{l} \max_{\tau,b'} \tau + b'q(b',s), \\ s.t. \; u(e^{def}(s) - \tau) + \beta E[V(b',s')|s] \geq u(e^{def}(s)) + \beta E\left[f_1(b,s')|s\right] \end{array} \right\}.$$

Finally, as the proposer can always guarantee themselves their reservation payoff (or the outside option in the case of the debtor) by proposing something that will not be accepted, it must be that

$$f_1(b,s) = \max\left\{ \begin{array}{c} \max_{\tau,b'} u(e^{def}(s) - \tau) + \beta E[V(b',s')|s] \\ s.t \;\; \tau + b'q(b',s) \geq \min\left\{b, \delta E[f_2(b,s')|s]\right\} \end{array}, u(e^{def}(s)) + \beta E\left[f_1(b,s')|s\right] \right\}$$

when the debtor proposes, and

$$f_2(b,s) = \max\left\{\min\left\{b, \begin{array}{c} \max_{\tau,b'} \tau + b'q(b',s) \\ s.t \;\; u(e^{def}(s)-\tau)+\beta E[V(b',s')|s] \geq u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right] \end{array}\right\}\right.$$
$$\left., \delta E\left[f_2(b,s')|s\right]\right\}$$

when the creditor proposes. But then $\hat{T}f = f$.

Second, suppose that $\hat{T}f = f$. We will construct a SSP outcome $((B\times S)^{\mu}, \mu)$ for which $f = v^{\mu}$. We construct $(B\times S)^{\mu}$ by noting that, if for a given (b,s) there exists (τ, b') such that

$$\begin{aligned} f_1(b,s) &= u(e^{def}(s)-\tau)+\beta E[V(b',s')|s] \\ f_2(b,s) &= \tau + b'q(b',s), \end{aligned}$$

then (b,s) is an agreement state and hence $(b,s)\in(B\times S)^{\mu}$. Then for that state we let

$$\mu(b,s) = (\tau, b').$$

Otherwise, we say $(b,s) \in (B\times S)\setminus(B\times S)^{\mu}$.

We need to show that the value of the outcome $((B\times S)^{\mu}, \mu)$, v^{μ}, is equal to f and that it is a SSP outcome. To show that the value of the outcome is v^{μ}, consider any state (b,s). Since $\hat{T}f = f$, for the non-proposing player we have

$$\begin{aligned} f_1(b,s) &= u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right], \\ f_2(b,s) &= \min\left\{b, \delta E\left[f_2(b,s')|s\right]\right\}, \end{aligned}$$

while for the proposing country we have

$$\hat{T}f_1(b,s) = \max\left\{\begin{array}{c} \max_{\tau,b'} u(e^{def}(s)-\tau)+\beta E[V(b',s')|s] \\ s.t \;\; \tau + b'q(b',s) \geq \delta E[f_2(b,s')|s] \end{array}, u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right]\right\},$$

with an analogous result for the creditor. If $\tau + b'q(b',s) = f_2(b,s)$, then $(b,s)\in(B\times S)^{\mu}$ by construction and

$$f_1(b,s) = u(e^{def}(s)-\tau)+\beta E[V(b',s')|s].$$

If $\tau + b'q(b', s) < f_2(b, s)$, then $(b, s) \notin (B \times S)^{\mu}$ and

$$f_1(b, s) = u(e^{def}(s)) + \beta E[f_1(b, s')|s].$$

but in Lemma 1 we showed that v^{μ} was the unique function satisfying these conditions. Hence $f = v^{\mu}$.

Finally, to show that $((B \times S)^{\mu}, \mu)$ is a SSP outcome, consider a strategy designed as follows: (i) if $(b, s) \in (B \times S)^{\mu}$, then propose $\mu(b, s)$, otherwise propose an outcome that delivers the other player strictly less than $v^{\mu}(b, s)$; (ii) accept any proposal as long as it delivers at least $v^{\mu}(b, s)$. To see that this is a subgame perfect equilibrium, consider a node at which a player has yet to propose. $\mu(b, s)$ delivers at least $v^{\mu}(b, s)$ by the previous result and so will be accepted. Moreover, as $\hat{T}v^{\mu} = v^{\mu}$, this proposal maximizes the payoff of the proposer subject to delivering this utility level. Hence a proposer cannot gain by deviating to any other proposal. Next, consider a node at which a proposal has been made. If the proposal gives strictly less than $v^{\mu}(b, s)$, the player can only lose by accepting it. If the proposal gives exactly $v^{\mu}(s)$, then by construction it also delivers exactly the reservation payoff of the agent, which is the value they expect from rejecting the offer. Hence, a one stage rejection of a proposal gives the same expected payoff. Familiar arguments show that by iterating on this argument we can rule out finite stage deviations, while boundedness and discounting rule out infinite deviations. ■

The fixed point of this operator forms the basis for our theoretical and numerical analysis of the bargaining problem below. In the next subsection we establish existence of an equilibrium bargain, and provide a sufficient condition under which this bargain is unique, by studying the properties of the $\hat{T}$ operator.

4.2 Existence and Uniqueness of Symmetric Subgame Perfect Equilibria

Next we can show that an SSP equilibrium exists, by showing that our $\hat{T}$ mapping is monotone. The proof of existence makes use of the following two Lemma's. In what follows, it is convenient to let $\mathcal{F}(B \times S)$ be the space of all functions mapping $B \times S$ into $\mathbb{R}^2$. Then define

$$\begin{aligned} b_{\min} &\equiv \min B, \\ b_{\max} &\equiv \max B, \\ V_{\max} &= \max_{(b,s,s')\in B\times S\times S} u(e(s)-b)+\beta V(b,s'), \\ V_{\min} &= \min_{s\in S} \frac{u\left(e^{def}(s)\right)}{1-\delta}, \end{aligned}$$

and let $\mathcal{B}(B \times S)$ be the subset of $\mathcal{F}(B \times S)$ that satisfy

$$\begin{aligned} \min_{(b,s)\in B\times S} f_1(b,s) &\geq V_{\min}, \\ \max_{(b,s)\in B\times S} f_1(b,s) &\geq V_{\max}, \\ \min_{(b,s)\in B\times S} f_2(b,s) &\geq b_{\min} \equiv \min B, \\ \max_{(b,s)\in B\times S} f_2(b,s) &\leq b_{\max} \equiv \max B. \end{aligned}$$

We endow $\mathcal{B}(B \times S)$ with the maximum (supremum) norm.

Lemma 3 *The operator $\hat{T}$ maps $\mathcal{B}(B \times S)$ into itself.*

Proof. To see that if $f \in \mathcal{B}(B \times S)$ then $\hat{T}f \in \mathcal{B}(B \times S)$, first consider the creditors continuation value function. Fix b. Then if s is such that the debtor proposes

$$\hat{T}_2(f_1,f_2)(b,s) = \min\{b, \delta E[f_2(b,s')|s]\} \in [b_{\min}, b_{\max}].$$

If s is such that the creditor proposes

$$\begin{aligned}\hat{T}_2\left(f_1,f_2\right)(b,s) &= \max\left\{\min\left\{\begin{array}{c}\max_{\tau,b'}\tau+b'q(b',s)\\ b,\ \ s.t\ \ u(e^{def}(s)-\tau)+\beta E[V(b',s')|s]\\ \geq u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right]\end{array}\right\},\delta E\left[f_2(b,s')|s\right]\right\}\\ &\leq \max\left\{b,\delta E\left[f_2(b,s')|s\right]\right\}\leq b_{\max},\end{aligned}$$

and

$$\begin{aligned}\hat{T}_2\left(f_1,f_2\right)(b,s) &= \max\left\{\min\left\{\begin{array}{c}\max_{\tau,b'}\tau+b'q(b',s)\\ b,\ \ s.t\ \ u(e^{def}(s)-\tau)+\beta E[V(b',s')|s]\\ \geq u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right]\end{array}\right\},\delta E\left[f_2(b,s')|s\right]\right\}\\ &\geq \delta E\left[f_2(b,s')|s\right]\geq b_{\min},\end{aligned}$$

since $b_{\min}\leq 0$.

Next consider the debtor's continuation value function. Fix b. Then if s is such that the creditor proposes

$$\begin{aligned}&\hat{T}_1\left(f_1,f_2\right)(b,s)\\ =&\ \max\left\{u\left(e^{def}(s)\right)+\beta E\left[f_1\left(b,s'\right)|s\right],\begin{array}{c}\max_{\tau,b'}u\left(e^{def}(s)-\tau\right)+\beta E\left[V\left(b',s'\right)|s\right]\\ s.t\ \ \tau+b'q\left(b',s\right)\geq b\end{array}\right\}\\ \leq&\ \max\left\{u\left(e^{def}(s)-b_{\min}\right)+\beta V_{\max},V_{\max}\right\}\leq V_{\max},\end{aligned}$$

and

$$\begin{aligned}\hat{T}_1\left(f_1,f_2\right)(b,s) &= \max\left\{u\left(e^{def}(s)\right)+\beta E\left[f_1\left(b,s'\right)|s\right],\begin{array}{c}\max_{\tau,b'}u\left(e^{def}(s)-\tau\right)+\beta E\left[V\left(b',s'\right)|s\right]\\ s.t\ \ \tau+b'q\left(b',s\right)\geq b\end{array}\right\}\\ &\geq u\left(e^{def}(s)\right)+\beta E\left[f_1\left(b,s'\right)|s\right]\geq V_{\min}.\end{aligned}$$

If s is such that the debtor proposes

$$\begin{aligned}&\hat{T}_1\left(f_1,f_2\right)(b,s)\\ =&\ \max\left\{\begin{array}{c}\max_{\tau,b'}u(e^{def}(s)-\tau)+\beta E[V(b',s')|s]\\ s.t\ \ \tau+b'q(b',s)\geq\min\left\{b,\delta E\left[f_2\left(b,s'\right)|s\right]\right\},\end{array},u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right]\right\}\\ \geq&\ u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right]\geq V_{\min},\end{aligned}$$

and

$$
\begin{aligned}
& \hat{T}_1\left(f_1, f_2\right)(b, s) \\
= & \max \left\{ \begin{array}{c} \max_{\tau, b'} u(e^{def}(s)-\tau)+\beta E[V(b', s')|s] \\ s.t \quad \tau+b' q(b', s) \geq \min\left\{b, \delta E\left[f_2\left(b, s'\right)|s\right]\right\}, \end{array} , u(e^{def}(s))+\beta E\left[f_1(b, s')|s\right] \right\} \\
\leq & \max \left\{V_{\max}, u\left(e^{def}(s)-b_{\min}\right)+\beta V_{\max}\right\} \leq V_{\max}.
\end{aligned}
$$

■

Lemma 4 *The operator $\hat{T}$ is monotone. That is, if there exists functions $f_1, f_1', f_2, f_2' \in \mathcal{F}(B \times S)$ such that $f_1 > f_1'$ and $f_2' > f_2$ then*

$$
\hat{T}_1(f_1, f_2) \geq \hat{T}_1(f_1', f_2') \quad \text{and} \quad \hat{T}_2(f_1, f_2) \leq \hat{T}_2(f_1', f_2').
$$

Proof. Take the functions f_1, f_1', f_2, f_2' as given. Fix b and consider a state s in which the debtor proposes. Then it follows immediately that the creditor's value satisfies

$$
\hat{T}_2(f_1, f_2)(b, s)=\min\left\{b, \delta E\left[f_2\left(b, s'\right)|s\right]\right\} \leq \min\left\{b, \delta E\left[f_2'\left(b, s'\right)|s\right]\right\}=\hat{T}_2(f_1', f_2')(b, s).
$$

For the debtor's value, we have

$$
\begin{aligned}
& \hat{T}_1\left(f_1, f_2\right)(b, s) \\
= & \max \left\{ \begin{array}{c} \max_{\tau, b'} u(e^{def}(s)-\tau)+\beta E[V(b', s')|s] \\ s.t \quad \tau+b' q(b', s) \geq \min\left\{b, \delta E\left[f_2\left(b, s'\right)|s\right]\right\}, \end{array} , u(e^{def}(s))+\beta E\left[f_1(b, s')|s\right] \right\} \\
\geq & \max \left\{ \begin{array}{c} \max_{\tau, b'} u(e^{def}(s)-\tau)+\beta E[V(b', s')|s] \\ s.t \quad \tau+b' q(b', s) \geq \min\left\{b, \delta E\left[f_2'\left(b, s'\right)|s\right]\right\}, \end{array} , u(e^{def}(s))+\beta E\left[f_1'(b, s')|s\right] \right\} \\
= & \hat{T}_1(f_1', f_2')(b, s).
\end{aligned}
$$

As this is true for all (b, s), monotonicity holds for this region of the state space.

Now consider s such that the creditor proposes. The debtor's value satisfies

$$\begin{aligned}
& \hat{T}_1(f_1, f_2)(b, s) \\
= \; & \max \left\{ u\left(e^{def}(s)\right) + \beta E\left[f_1(b, s')|s\right], \begin{array}{c} \max_{\tau, b'} u\left(e^{def}(s) - \tau\right) + \beta E\left[V(b', s')|s\right] \\ s.t \;\; \tau + b'q(b', s) \geq b \end{array} \right\} \\
\geq \; & \max \left\{ u\left(e^{def}(s)\right) + \beta E\left[f_1'(b, s')|s\right], \begin{array}{c} \max_{\tau, b'} u\left(e^{def}(s) - \tau\right) + \beta E\left[V(b', s')|s\right] \\ s.t \;\; \tau + b'q(b', s) \geq b \end{array} \right\} \\
= \; & \hat{T}_2(f_1', f_2')(b, s).
\end{aligned}$$

Similarly, the creditor's value satisfies

$$\begin{aligned}
& \hat{T}_2(f_1, f_2)(b, s) \\
= \; & \max \left\{ \min \left\{ b, \begin{array}{c} \max_{\tau, b'} \tau + b'q(b', s) \\ s.t \;\; u(e^{def}(s) - \tau) + \beta E[V(b', s')|s] \\ \geq u(e^{def}(s)) + \beta E\left[f_1(b, s')|s\right] \end{array} \right\}, \delta E\left[f_2(b, s')|s\right] \right\} \\
\leq \; & \max \left\{ \min \left\{ b, \begin{array}{c} \max_{\tau, b'} \tau + b'q(b', s) \\ s.t \;\; u(e^{def}(s) - \tau) + \beta E[V(b', s')|s] \\ \geq u(e^{def}(s)) + \beta E\left[f_1(b, s')|s\right] \end{array} \right\}, \delta E\left[f_2'(b, s')|s\right] \right\} \\
\leq \; & \max \left\{ \min \left\{ b, \begin{array}{c} \max_{\tau, b'} \tau + b'q(b', s) \\ s.t \;\; u(e^{def}(s) - \tau) + \beta E[V(b', s')|s] \\ \geq u(e^{def}(s)) + \beta E\left[f_1'(b, s')|s\right] \end{array} \right\}, \delta E\left[f_2'(b, s')|s\right] \right\} \\
= \; & \hat{T}_2(f_1', f_2')(b, s),
\end{aligned}$$

where the last inequality comes from the fact that $f_1' \leq f_1$ which loosens the constraint on the creditor's maximization problem and thus weakly increases the value of the program. ■

The proof of existence then follows by applying the $\hat{T}$ operator to a suitable initial f^0 within the space $\mathcal{B}(B \times S)$.

Theorem 5 *An SSP equilbrium exists.*

Proof. Choose $f^0 = (f_1^0, f_2^0)$ such that for all (b, s), $f_1^0(b, s) = V_{\max}$ and $f_2^0(b, s) = b_{\min}$and successively apply the operator $\hat{T}$ to obtain the sequence of functions $\{f^n\}_{n=0}^{\infty}$ where $f^{n+1} = \hat{T} f^n$. By Lemma 4 $\hat{T}$ is monotone, and by Lemma 3 $\hat{T}$ maps $\mathcal{B}(B \times S)$ into itself, so that this is a monotone sequence of functions in $\mathcal{B}(B \times S)$. Hence, the sequence converges to a SSP equilibrium values and by Theorem 2 and Lemma 1 there exists a SSP equilibrium. ■

The uniqueness of the values of the equilibrium bargain can be easily established if we can show that $\hat{T}$ is a contraction mapping. However, as in many multi-agent problems, this is not straightforward. The difficulty is that changes in one agent's continuation value function will affect the result of the operator on the other agents continuation value function, because continuation values act as constraints on the proposals that will be made and accepted. In our problem, this issue is magnified by the fact that continuation values for the two agents are measured in different units; changes in creditor values, which are measured in terms of real resources, will have effects on debtor values, measured in terms of utility, and vice versa.

The following theorem states a condition that is sufficient to prove uniqueness, by imposing bounds on the rate at which resources can be transformed into utility, and the rate at which utility can be transformed into resources. As a consequence of the fact that we have imposed few restrictions on the shape of the V and q functions, the condition is stated in terms of bounds on the slope of the utility function of the debtor; a weaker sufficient condition can be obtained under stronger conditions on the shapes of V and q. In our numerical work below, as in much of the quantitative literature on sovereign debt and default, we focus on discount factors for the country that are substantially less than one, reflecting political economy problems in developing countries that lead to impatient policy making. For such parameter values, the sufficient condition is typically satisfied.

Theorem 6 *If there exists $K_L > \beta$ and $K_U < 1/\delta$ such that*

$$\frac{1}{K_L} \leq u'(c) \leq K_U,$$

then the SSP equilibrium values are unique.

Proof. Let $f^1 = (f_1^1, f_2^1)$ and $f^2 = (f_1^2, f_2^2)$ be elements of $\mathcal{B}(B \times S)$. To establish the result, we need to show that there exists a $\gamma \in (0,1)$ such that

$$\begin{aligned}
& ||\hat{T}f^1 - \hat{T}f^2||_\infty \\
= & \max_{(b,s)\in B\times S} \left\{ \max\left\{ \left| \left(\hat{T}f_1^1\right)(b,s) - \left(\hat{T}f_1^2\right)(b,s)\right|, \left| \left(\hat{T}f_2^1\right)(b,s) - \left(\hat{T}f_2^2\right)(b,s)\right| \right\}\right\} \\
\leq & \gamma \max_{(b,s)\in B\times S} \left\{ \max\left\{ \left| f_1^1(b,s) - f_1^2(b,s)\right|, \left| f_2^1(b,s) - f_2^2(b,s)\right| \right\}\right\} \\
\leq & \gamma ||f^1 - f^2||_\infty.
\end{aligned}$$

The argument varies according to whether the outside offer is taken, no proposal is accepted, or a proposal is accepted.

First, fix (b,s) and consider the case in which s is such that the debtor proposes. If the outside option is taken for both f^1 and f^2, then we have

$$\left| \left(\hat{T}f^1\right)(b,s) - \left(\hat{T}f^2\right)(b,s)\right| = 0,$$

since the creditor's payoff is b, and the debtor's payoff solves

$$\begin{gathered}
\max_{\tau,b'} u(e^{def}(s) - \tau) + \beta E[V(b',s')|s] \\
s.t \quad \tau + b'q(b',s) \geq b,
\end{gathered}$$

neither or which depends on the continuation values f^1 and f^2.

If no proposal is accepted for both f^1 and f^2, then we have

$$\left| \left(\hat{T}f_2^1\right)(b,s) - \left(\hat{T}f_2^2\right)(b,s)\right| = \left|\delta E\left[f_2^1(b,s')|s\right] - \delta E\left[f_2^2(b,s')|s\right]\right| \leq \delta ||f_2^1 - f_2^2||_\infty,$$

for the creditor's continuation value function, and

$$
\begin{aligned}
&\left|\left(\hat{T}f_1^1\right)(b,s)-\left(\hat{T}f_1^2\right)(b,s)\right| \\
= &\left|u(e^{def}(s))+\beta E\left[f_1(b,s')|s\right]-u(e^{def}(s))-\beta E\left[f_1(b,s')|s\right]\right| \\
= &\beta\left|E\left[f_1(b,s')|s\right]-E\left[f_1(b,s')|s\right]\right| \\
\leq &\beta\left\|f_1^1-f_1^2\right\|_\infty .
\end{aligned}
$$

for the debtor's continuation value function.

If a proposal is accepted for both f^1 and f^2, consider first the case in which s is such that the debtor proposes. In this case, the creditor's continuation values satisfy

$$
\left|\left(\hat{T}f_2^1\right)(b,s)-\left(\hat{T}f_2^2\right)(b,s)\right|=\left|\delta E\left[f_2^1(b,s')|s\right]-\delta E\left[f_2^2(b,s')|s\right]\right|\leq\delta||f_2^1-f_2^2||_\infty .
$$

Using this fact, the debtor's continuation values satisfy

$$
\begin{aligned}
&\left|\left(\hat{T}f_1^1\right)(b,s)-\left(\hat{T}f_1^2\right)(b,s)\right| \\
= &\left|\begin{array}{c}\max_{\tau,b'} u\left(e^{def}(s)-\tau\right)+\beta E\left[V(b',s')|s\right] \\ s.t\ \ \tau+b'q(b',s)\geq\delta E\left[f_2^1(b,s')|s\right]\end{array}\right. \\
&\left.-\begin{array}{c}\max_{\tau,b'} u\left(e^{def}(s)-\tau\right)+\beta E\left[V(b',s')|s\right] \\ s.t\ \ \tau+b'q(b',s)\geq\delta E\left[f_2^2(b,s')|s\right]\end{array}\right| \\
\leq &\left|\begin{array}{c}\max_{\tau,b'} u\left(e^{def}(s)-\tau\right)+\beta E\left[V(b',s')|s\right] \\ s.t\ \ \tau+b'q(b',s)\geq\delta E\left[f_2^2(b,s')|s\right]+\delta||f_2^1-f_2^2||_\infty\end{array}\right. \\
&\left.-\begin{array}{c}\max_{\tau,b'} u\left(e^{def}(s)-\tau\right)+\beta E\left[V(b',s')|s\right] \\ s.t\ \ \tau+b'q(b',s)\geq\delta E\left[f_2^2(b,s')|s\right]\end{array}\right| .
\end{aligned}
$$

Now suppose that (τ^2, b'^2) attain the maximum for f_2^2. Then exploiting the fact that U is defined over negative consumptions and that its slope is bounded we can we define a feasible $\hat{\tau}$ such that

$$
\hat{\tau}=\tau^2+\delta\left\|f_2^1-f_2^2\right\|_\infty ,
$$

yielding

$$
\begin{aligned}
& \left|\left(\hat{T}f_1^1\right)(b,s)-\left(\hat{T}f_1^2\right)(b,s)\right| \\
\leq\ & \left|u\left(e^{def}(s)-\tau^2+\delta\left\|f_2^1-f_2^2\right\|_\infty\right)+\beta E\left[V\left(b^2,s'\right)|s\right]-u\left(e^{def}(s)-\tau^2\right)\right. \\
& \left.-\beta E\left[V\left(b^2,s'\right)|s\right]\right| \\
\leq\ & \left|u\left(e^{def}(s)-\tau^2\right)+u'\left(e^{def}(s)-\tau^2\right)\beta\left\|f_2^1-f_2^2\right\|_\infty-u\left(e^{def}(s)-\tau^2\right)\right| \\
\leq\ & \delta K_U\left\|f_2^1-f_2^2\right\|_\infty .
\end{aligned}
$$

Next consider the case in which s is such that the creditor proposes. In this case, the debtor's continuation values satisfy

$$
\begin{aligned}
& \left|\left(\hat{T}f_1^1\right)(b,s)-\left(\hat{T}f_1^2\right)(b,s)\right| \\
=\ & \left|\beta E\left[f_1^1(b,s')|s\right]-\beta E\left[f_1^2(b,s')|s\right]\right|\leq\beta||f_1^1-f_1^2||_\infty.
\end{aligned}
$$

Using this fact, the creditor's continuation values satisfy

$$
\begin{aligned}
& \left|\left(\hat{T}f_2^1\right)(b,s)-\left(\hat{T}f_2^2\right)(b,s)\right| \\
=\ & \left|\begin{array}{c}\max_{\tau,b'}\tau+b'q(b',s)\\ s.t\ \ u(e^{def}(s)-\tau)+\beta E[V(b',s')|s]\\ \geq u(e^{def}(s))+\beta E\left[f_1^1(b,s')|s\right]\end{array}-\begin{array}{c}\max_{\tau,b'}\tau+b'q(b',s)\\ s.t\ \ u(e^{def}(s)-\tau)+\beta E[V(b',s')|s]\\ \geq u(e^{def}(s))+\beta E\left[f_1^2(b,s')|s\right]\end{array}\right| \\
\leq\ & \left|\begin{array}{c}\max_{\tau,b'}\tau+b'q(b',s)\\ s.t\ \ u(e^{def}(s)-\tau)+\beta E[V(b',s')|s]\\ \geq u(e^{def}(s))+\beta E\left[f_1^1(b,s')|s\right]\end{array}\right. \\
& \left.-\begin{array}{c}\max_{\tau,b'}\tau+b'q(b',s)\\ s.t\ \ u(e^{def}(s)-\tau)+\beta E[V(b',s')|s]\\ \geq u(e^{def}(s))+\beta E\left[f_1^1(b,s')|s\right]+\beta\left\|f_1^1-f_1^2\right\|_\infty\end{array}\right|.
\end{aligned}
$$

Now suppose that (τ^1,b'^1) attain the maximum for f_1^1. Then there exists a $\hat{\tau}$ such that

$$
\left|\hat{\tau}-\tau^1\right|\leq\beta K_L\left\|f_1^1-f_1^2\right\|_\infty ,
$$

and that $(\hat{\tau}, b'^2)$ is feasible for f_1^1 and so

$$\left|\left(\hat{T}f_2^1\right)(b,s) - \left(\hat{T}f_2^2\right)(b,s)\right| \leq \beta \frac{1}{K_L} \left\|f_1^1 - f_1^2\right\|_\infty .$$

It remains to consider cases that involve combinations of the outside option, no proposal being accepted, and a proposal being accepted. Suppose the outside option is taken for one of the f^i and no proposal is accepted for f^{-i}. The argument is analogous regardless of whether the debtor proposes or the creditor proposes at s. Without loss of generality we can order the creditor's continuation value functions such that

$$\begin{aligned}
\left|\left(\hat{T}f_2^1\right)(b,s) - \left(\hat{T}f_2^2\right)(b,s)\right| &= \left|b - \delta E\left[f_2^2\left(b,s'\right)|s\right]\right| \\
&\leq \left|\delta E\left[f_2^1\left(b,s'\right)|s\right] - \delta E\left[f_2^2\left(b,s'\right)|s\right]\right| \\
&\leq \delta||f_2^1 - f_2^2||_\infty,
\end{aligned}$$

while for the debtor, if we define

$$V^{oo}(b,s) = \begin{array}{c} \max_{\tau,b'} u(e^{def}(s) - \tau) + \beta E[V(b',s')|s] \\ s.t \;\; \tau + b'q(b',s) \geq b, \end{array}$$

we have

$$\begin{aligned}
& \left|\left(\hat{T}f_1^1\right)(b,s) - \left(\hat{T}f_1^2\right)(b,s)\right| \\
=\; & \left|V^{oo}(b,s) - u(e^{def}(s)) + \beta E\left[f_1^2(b,s')|s\right]\right| \\
\leq\; & \left|u\left(e^{def}(s)\right) + \beta E\left[f_1^2\left(b,s'\right)|s\right] - u(e^{def}(s)) - \beta E\left[f_1^1(b,s')|s\right]\right| \\
\leq\; & \beta\left\|f_1^1 - f_1^2\right\|_\infty .
\end{aligned}$$

where the first inequality follows from the fact that the debtor did not take the outside option for f^2 and the fact that the value of the outside option is independent of the continuation values.

Now suppose the outside option is taken for one of the f^i and a proposal is accepted for f^{-i}. If s is such that the debtor proposes, then the argument for the creditor is the same as in the previous case since they earn their autarky value from an accepted proposal. For the debtor, we have

$$
\begin{aligned}
& \left|\left(\hat{T} f_1^1\right)(b, s)-\left(\hat{T} f_1^2\right)(b, s)\right| \\
= & \left|\begin{array}{c}
\max_{\tau, b'} u\left(e^{def}(s)-\tau\right)+\beta E\left[V\left(b', s'\right)|s\right] \\
s.t \quad \tau+b' q\left(b', s\right) \geq \delta E\left[f_2^1\left(b, s'\right)|s\right]
\end{array}-V^{oo}(b, s)\right| \\
\leq & \left|\begin{array}{c}
\max_{\tau, b'} u\left(e^{def}(s)-\tau\right)+\beta E\left[V\left(b', s'\right)|s\right] \\
s.t \quad \tau+b' q\left(b', s\right) \geq \delta E\left[f_2^1\left(b, s'\right)|s\right]
\end{array}-\begin{array}{c}
\max_{\tau, b'} u\left(e^{def}(s)-\tau\right)+\beta E\left[V\left(b', s'\right)|s\right] \\
s.t \quad \tau+b' q\left(b', s\right) \geq \delta E\left[f_2^2\left(b, s'\right)|s\right]
\end{array}\right| \\
\leq & \delta K_U\left\|f_2^1-f_2^2\right\|_{\infty},
\end{aligned}
$$

where the first inequality follows from the fact that the debtor did not take the outside option for f^2. If s is such that the creditor proposes, the argument for the debtor's continuation value function is the same as in the previous case because the debtor receives their autarky value from an accepted proposal. For the creditor, the result follows from an argument similar to the debtor proposer case.

Finally, consider the case where no agreement occurs for f^1 and an agreement occurs for f^2. Non-proposers receive their autarky values in both cases, implying no difference in continuation value functions under the $\hat{T}$ operator. For the proposer, the fact that no agreement is chosen over agreement for f^2 means we can apply the same argument as in the previous case.

Since the result holds for arbitrary (b, s), the operator T is a contraction with modulus

$$\gamma=\max\left\{\delta, \beta, \delta K_U, \beta / K_L\right\}.$$

■

5. SOLUTION TO BORROWING MODEL

In the previous section, we characterized the solution to the debt restructuring bargaining problem taking as given the value to the country from reaccessing capital markets with new debt b', $E[V(b', s')|s]$, and the value of new debt to creditors $q(b', s)$. In this section, we take as given the solution to the bargaining model and hence the value to the country and the creditor from being in default, and then characterize the solution to the borrowing problem. That is, we take as given the $N_e \times N_b$ vectors of payoffs to the country, $\tilde{V}^D(b, s)$, and the creditor, $\tilde{W}(b, s)$, in default, that are elements of $\mathcal{B}(B \times S)$.

We then go on to consider the equilibrium of our entire model and provide a set of conditions under which an equilibrium exists. Finally we illustrate the time path for two possible defaults, before returning to the application.

5.1 The Default and Repayment Decision

The solution of the borrowing problem is established as the composition of two operators. The first takes a value to the country from default and an equilibrium bond price function, and then solves the country's problem to obtain a value to the country for acces to capital markets, and a default policy function, which is a selection from a default policy correspondence. The second takes the default policy function and combines it with the value to the creditors from default to obtain a new bond price function. Iterating on these operators produces a fixed point.

Lemma 7 *Given* $\left(\tilde{V}^D(b, s), \tilde{W}(b, s)\right) \in \mathcal{B}(B \times S)$ *and* $q(b, s) \in \mathcal{Q}(B \times S)$, *there exists a solution to the country's borrowing problem,* $V(b, s)$.

Proof. Let $\mathcal{G}(B \times S)$ be the space of all real valued functions on $B \times S$, and for any $f \in \mathcal{G}(B \times S)$ define the operator T^V by

$$\left(T^V f\right)(b,s) = \max \left\{ \begin{array}{c} \max_{c,b' \in B} U(c) + \beta \sum_{s' \in S} \pi(s'|s) f(b', s'), \\ s.t.\ c - q(b', s) b' \leq e(s) + b. \end{array}, \tilde{V}^D(b,s) \right\}.$$

First, we show that the operator T^V is monotone. Let $f^1, f^2 \in \mathcal{G}(B \times S)$ such that $f^1 \geq f^2$. Then for all (b,s)

$$\begin{aligned} \left(T^V f^1\right)(b,s) &= \max \left\{ \begin{array}{c} \max_{c,b' \in B} U(c) + \beta \sum_{s' \in S} \pi(s'|s) f^1(b', s'), \\ s.t.\ c - q(b', s) b' \leq e(s) + b. \end{array}, \tilde{V}^D(b,s) \right\} \\ &\geq \max \left\{ \begin{array}{c} \max_{c,b' \in B} U(c) + \beta \sum_{s' \in S} \pi(s'|s) f^2(b', s'), \\ s.t.\ c - q(b', s) b' \leq e(s) + b. \end{array}, \tilde{V}^D(b,s) \right\} \\ &\geq \left(T^V f^2\right)(b,s). \end{aligned}$$

Next, define f^0 by

$$f^0(b,s) = \max \left\{ \max_{(b,s) \in B \times S} \frac{u(e(s) - b)}{1 - \delta}, \max_{(b,s) \in B \times S} \tilde{V}^D(b,s) \right\},$$

for all (b,s), and construct the sequence of functions $\{f^n\}_{n=0}^{\infty}$ where $f^{n+1} = T^V f^n$. Then this is a monotonically decreasing sequence of functions in $\mathcal{G}(B \times S)$ that is bounded below by $\tilde{V}^D(b,s) \in \mathcal{G}(B \times S)$. Hence, the sequence converges to an element in $\mathcal{G}(B \times S)$. ■

Lemma 8 *Given* $\left(\tilde{V}^D(b,s), \tilde{W}(b,s)\right) \in \mathcal{B}(B \times S)$, *there exists an equilibrium bond price function* $q(b,s) \in \mathcal{Q}(B \times S)$.

Proof. For any $g^n \in \mathcal{Q}(B \times S)$, define the operator T^q as follows. First, given g, apply the operator T^V (which is defined for a given g) until convergence to V^n with associated $\left(V^R\right)^n$. Then define

$$\phi^n(b,s) = \left\{ \begin{array}{ll} 1 & \text{if } \tilde{V}^D(b,s) > \left(V^R\right)^n(b,s) \\ 0 & \text{if } \tilde{V}^D(b,s) \leq \left(V^R\right)^n(b,s) \end{array} \right.,$$

which embodies the behavioral assumption that when indifferent between default and repayment the country always repays, from which can be constructed the default probability

$$p^n(b,s) = \sum_{b\in B, s'\in S} \phi^n(b,s')\,\pi(s'|s),$$

and a new bond price function

$$g^n(b,s) = \frac{1 - p(b,s) + p(b,s)\sum_{s'\in S}\pi(s'|s)\,\tilde{W}(b,s')/b}{1+r^w},$$

which is an element of $\mathcal{Q}(B\times S)$ given the bounds on $\tilde{W}(b,s)$.

Then define the sequence $\{g^n\}_{n=0}^{\infty}$ by applying T^q successively from the initial $g^0 = 1/(1+r^w)$. To see that this is a monotone sequence in $\mathcal{Q}(B\times S)$, note that $g^1 \le g^0$ and moreover that $\phi^n(b,s) = 0$ whenever $b \le 0$. Hence, the interest rate on borrowings is increasing at each stage, while the interest rate on savings is unchanged, and consequently the fixed points of the associated T^V operators are ordered. But this produces an ordered sequence of default probabilities p^n and, given our restriction on $\tilde{W}(b,s)$, a monotonically decreasing sequence of g^n. As this sequence is bounded below by zero, it converges to a fixed point in $\mathcal{Q}(B\times S)$. ■

Given the results of the above two Lemma's, it is tempting to try to prove existence of an equilibrium for our entire model by iterating successively on the T^V, T^q and $\hat{T}$ operators. However, this approach need not converge. Specifically, although iterating on the T^V and T^q operators produces a monotone operator, when combined with the bargaining operator, the compounded operator need not be monotone. Intuitively, it can be the case that a high value to the creditor in default, and a low value to debtor, leads to a high bond price, which in turn leads to a high value to the country from repayment. This high value to repayment can lead to a high value from default, which then leads to a low bond price in the next iteration. That is, we cannot rule out cycles in the successive application of these operators.

In the next section, we describe an alternative method for proving existence.

5.2 Existence of Equilibrium

In this section, we establish the existence of a recusive equilibrium for our economy. First, we define an equilibrium for our economy.

Definition 9 *An* ***equilibrium for our borrowing economy*** *is a value function for the country from borrowing $V(b,s)$, a value function for the country in default $V^D(b,s)$, a value function for the creditor in default $W(b,s)$ and a bond price function $q(b,s)$ such that:*

A Given the bond price function $q(b,s)$ and the value to the country from reaccessing capital markets $V(b,s)$, the country and the creditor optimally bargain over reaccess to financial markets. That is, V^D and W are fixed points of the **inside default** operator $\hat{T}$;

B Given the value to the country and from default $V^D(b,s)$, and the bond price $q(b,s)$, the country makes optimal borrowing and default decisions. That is, $V(b,s)$ is a fixed point of T^V with associated default policy correspondence $\Phi(b,s)$

C Given the payoff to the creditor in default W and the optimal default policy correspondence, the bond price function $q(b,s)$ satisfies the no arbitrage condition for creditors. That is, $q(b,s)$ is a fixed point of the operator T^q..

The latter two conditions may equivalently be written as: Given $V^D(b,s)$ and $W(b,s)$, $V(b,s)$ and $q(b,s)$ are a fixed point of the **outside default** operator, which is the composition of the T^V and T^q operators.

We prove existence by using the operators defined above to construct a new mapping from the space of value functions for the country and creditor in default, and the space of bond price functions, into itself, and establishing that it possesses a fixed point. Specifically, define the mapping H from $\mathcal{B}(B \times S) \times \mathcal{Q}(B \times S)$ into itself as follows. First, given V^D, W and q, iterate on the outside default operator to find a new bond price function $q'(b,s)$. Second, given V^D and q, iterate on the T^V operator to produce a value function V. Then, given q and this V, iterate on the $\hat{T}$ operator to find a fixed point V^D and W. We establish that the combination of these operators defines an upper hemi-continuous correspondence with non-empty and convex values. Then, noting that $\mathcal{B}(B \times S) \times \mathcal{Q}(B \times S)$ is a compact and convex space of functions, the result then follows by application of the Kakutani-Fan-Glicksberg fixed point theorem.

Theorem 10 *There exists an equilibrium of our borrowing economy.*

Proof. Let $q \in \mathcal{Q}(B \times S)$ and $\left(V^D, W\right) \in \mathcal{B}(B \times S)$. We construct the first part of our mapping, $\mathcal{H}_1\left(V^D, q\right)$ as follows. Fix (b,s) and think of the $q(b',s)$ and $V^D(b,s)$ as a set of N_b+1 parameters for the country's borrowing problem. Let $C(X)$ be the set of continuous and bounded functions defined on $X = [0, 1/(1+r^w)]^{N_b+1} \times [V\min, V_{\max}]$. Let $f \in \mathcal{C}(X)$ and define the operator $\hat{T}^V$ by

$$\left(\hat{T}^V\right) f = \max\left\{\max_{b' \in B} u\left(e(s) - b + b'q(b',s)\right) + \beta E\left[V(b',s')\,|s\right], \tilde{V}^D(b,s)\right\}.$$

Next define $H_1\left(T^V, q\right)$ as the fixed point of the bargaining operator, given a default value of T^V and a bond price of q.

The finiteness of B ensures that a solution to the country's borrowing problem exists, and that it is bounded, while the Theorem of the Maximum implies that $\left(\hat{T}^V\right) f$ is continuous in x. For any $f^1, f^2 \in \mathcal{C}(X)$ analogues of the arguments provided above ensure that the fixed points of the bargaining operator defined on

$\mathcal{C}(X)$ are also continuous in X. Select the largest such fixed point. Then the mapping $\mathcal{H}_1\left(V^D, q\right)(b, s)$ is a continuous (and hence upper hemi-continuous) single valued, and hence compact and convex valued, correspondence. From this, we can construct the product correspondence

$$\mathcal{H}_1\left(V^D, q\right) = \prod_{(b,s)\in B\times S} \mathcal{H}_1\left(V^D, q\right)(b, s).$$

By Theorem 17.28 of Aliprantis and Border (2006), this product correspondence is continuous and compact valued.

Now consider the second part of our mapping $\mathcal{H}_2\left(V^D, W, q\right)$ defined as follows. First, think of the $q(b', s)$, $V^D(b, s)$ and $W(b, s)$ as a finite set of parameters for the country's borrowing problem, with each $q(b', s)$ belonging to the compact interval $[0, 1/(1 + r^w)]$, each $V^D(b, s)$ belonging to $[V_{\min}, V_{\max}]$, and each $W(b, s)$ belonging to $[b_{\min}, b_{\max}]$. Let $\mathcal{C}(X)$ be the space of all continuous functions defined on

$$X = [0, 1/(1 + r^w)]^{N_b\times N_e} \times [V_{\min}, V_{\max}]^{N_b\times N_e} \times [b_{\min}, b_{\max}]^{N_b\times N_e}.$$

Let $f \in \mathcal{C}(X)$ and define the operator $\hat{T}^V$ be defined by

$$\left(\hat{T}^V\right) f = \max\left\{\max_{b'\in B} u\left(e(s) - b + b'q(b', s)\right) + \beta E\left[V(b', s')\,|s\right], \tilde{V}^D(b, s)\right\}.$$

As above, the fixed point V is continuous on X; the calculations also define the function $V^R(b, s)$.

Define the default indicator correspondence

$$\Phi(b, s) = \begin{cases} 1 & \text{if } \tilde{V}^D(b, s) > V^R(b, s) \\ 0 & \text{if } \tilde{V}^D(b, s) < V^R(b, s) \\ [0, 1] & \text{if } \tilde{V}^D(b, s) = V^R(b, s) \end{cases}.$$

From this we can define a default probability correspondence, $P(b', s)$, as the set of all $p(b', s)$ constructed as

$$p(b', s) = \sum_{s'\in S} \phi(b', s')\,\pi(s'|s),$$

for some $\phi(b', s'; x) \in \Phi(b', s'; x)$., Hence, for any fixed (b', s) we can define the bond price correspondence from points in X to $[0, 1/(1 + r^w)]$ as

$$\mathcal{H}_2\left(V^D, W, q\right)(b', s) = \left\{ y : y = \frac{1 - p + p\sum_{s' \in S} \pi(s'|s)\, \tilde{W}(b', s')/b}{1 + r^w} \text{ for some } p \in P(b', s) \right\},$$

where $\tilde{W}(b', s')$ was defined above.

It is straightforward to show that for (b', s) and $\left(V^D, W, q\right)$ fixed, this is a closed interval contained in $[0, 1]$. Hence, it is compact valued. A straightforward adaptation of App Lemma 8 from Chaterjee, Corbae, Nakajima and Rios-Rull (2002) shows that it is also upper-hemi continuous. Therefore, viewed as a correspondence from points in X to $[0, 1/(1 + r^w)]$ this is upper-hemi continuous. Then for any $\left(V^D, W, q\right)$, we can define the product correspondence

$$\mathcal{H}_2\left(V^D, W, q\right) = \prod_{(b,s) \in B \times S} \mathcal{H}_2\left(V^D, W, q\right)(b, s).$$

By Theorem 17.28 of Aliprantis and Border (2006), this product correspondence is continuous and compact valued.

Finally, form

$$\mathcal{H}\left(V^D, W, q\right) = \left[\mathcal{H}_1\left(V^D, q\right), \mathcal{H}_2\left(V^D, W, q\right)\right].$$

By Theorem 17.23 of Aliprantis and Border (2006), $\mathcal{H}$ is upper hemi-continuous. Using the fact that $\mathcal{H}_1$ is single valued, it is also straightforward to show that it is convex valued. Hence, by Kakutani's fixed point theorem there exists a fixed point of $\mathcal{H}$.

Using the fixed points for q^* and V^{D*}, we can then iterate to convergence to find V^*. The collection V^*, V^{D*}, W^* and q^* satisfies the definition for an equilibrium of our borrowing economy, and hence there exists an equilibrium for our borrowing economy. ■

6. CALIBRATION AND RESULTS

In this section, we present results for several calibrated versions of the model. Each version varies only in the calibration of the bargaining power process for the debtor and creditor, and are used to illustrate numerically some comparative statics of the model. We also use them to build intuition for the elements of the model that produce delay. We then present our benchmark case in which the parameters of the model governing bargaining power are estimates from the relationship between default and output observed in the data. The model is then assessed according to it's ability to match the other facts discussed in the introduction. We find that the above model can quantitatively account for 66% of the delays experienced by countries in default, and captures five of the six facts listed above.

6.1 Calibration

To construct model output, we consider the evolution of a single economy over a large period of time. We simulate the economy from an arbitrary initial condition for 11000 periods and drop the first thousand periods to compile moments from the table. This limits the ability of the initial condition to affect outcomes. From these simulations we construct a series of moments from the model and compare them to the previously derived moments in the data. We treat model variables identically to data variables.

There are some parameter values we hold constant for every experiment. For these parameters, we make similar choices to other authors in the literature. Our experiments have much in common with those Arellano (2007), Yue (2007), Aguiar and Gopinath (2006), and Tomz and Wright (2007). Thus the majority of our model parameters are standard. These parameters include the discount factor, the international lending rate, the curvature of the utility function, the output penalty from default, and the persistence and standard deviation of the income process. We fix the length of a period to a quarter. The rest of our parameter choices are summarized in the table below and discussed subsequently. By selecting standard variables we preserve comparability wherever possible. Our parameter choices are listed in Table 12.

Some words about the sources for these parameter choices are in order. The debtor's discount factor is chosen to produce an annual discount factor of 0.8. We pick this parameter to match Yue (2007), Aguiar and Gopinath (2006) and Tomz and Wright (2007). The average world interest rate is set to 1% per quarter. Hence debtor nations are relatively more impatient than creditors, which provides an incentive for countries to borrow to tilt their consumption profile, and is designed to capture political economy considerations that lead governments in developing countries to act impatiently.The degree of risk aversion is two which is standard.

The income process is assumed to follow a log normal AR(1) process. We choose the parameters of that process to match Arellano (2007) as our goal is to preserve comparability on as many dimensions as possible.[3] The literature has typically assumed that a country in default loses a fraction of its endowment, which is designed to capture the direct economic effects of a default. This is a difficult parameter to calibrate because without a natural experiment, it is difficult to isolate the endogenous and exogenous components of the fall in output. We follow the literature, which assumes a loss of output of two percent per year while in default punishment.

Finally, we turn to a detailed description of our calibration of the relative bargaining power of debtors and creditors, which is summarized in the evolution of the probability that a given party will have the right to propose a settlement in the fu-

[3] We use different output penalties than Arellano (2007). Our definition of comparability is with our output costs. With the alternative output penalties we are able to generate higher debt to GDP ratios and default rates than with the paper in the literature.

ture. These parameters are unique to our analysis and are not based on observable criteria. We therefore pursue two strategies. First, we consult the international relations literature in order to construct cases in which the evolution of the proposers' probabilities capture key theories of international bargaining. This strategy allows us to demonstrate the range of outcomes that are possible within our model. Second, below we estimate this process using a simulated method of moments, which allows us to make detailed quantitative predictions.

We begin with our numerical comparative statics, for which we consider four differing styles of bargaining power regimes. We begin with a regime in which the likelihood that a party will be able to make the proposal in the next period is i.i.d; that is the choice of the future proposer does not depend on any features of the current state. In this regime, bargaining power is essentially constant over time and unrelated to economic conditions, with relative bargaining power captured by the probability that a given party is the proposer. The second regime introduces persistence in the choice of the proposer, so that the choice of future proposers depends on the identity of the current proposer. In this case, there will be cycles in bargaining power, although unlike the previous case they need not be related to changes in economic conditions.

The third and fourth regimes consider theories of bargaining where the strength of the two agents depends on economic conditions in the country. In third regime, which we refer to as "strength through weakness", the likelihood that the country is able to make offers in the future is higher when output is low, so that the debtors bargaining power is greatest when the economy is weak. The fourth regime, which we refer to as "strength through strength", gives the debtor country more bargaining power when output is high. These two regimes were patterned after theories from the international relations and political science literature. Under strength through weakness we consider the possibility that with poor economic performance the politicians negotiating the policy are too domestically vulnerable to propose significant concessions to its lenders, which acts as a form of bargaining power. Under strength through strength we assume that strong economic performance insulates a political leader from pressures from those domestic constituents and foreign allies who desire a quick agreement.

The implementation of these regimes is straightforward. For an i.i.d regime we assume the probability of proposing in the next period is independent of the state. We consider two i.i.d regimes, that vary according to whether or not the debtor or the creditor has more bargaining power.[4] For the persistent case, we simply assume that the probability of proposing in the next period depends solely on the current proposer. Numerically, this probability is 0.98 if an agent proposed in the previous period and 0.02 if the agent received the offer. The other two potential regimes: strength through weakness and strength through strength can be simulated in many different ways. To capture either strength through strength or strength through weakness we assume that the probability of proposing in the next period depends upon the relationship between the current realization of income and the mean of the process governing income. If income is above the mean, the side that for whom the bargaining power is associated with strength proposes with a probability greater than one half (95%). If income is below, that side proposes with probability less than one half (5%).

As shown in Table 14 below, the choice of regime has a significant effect on outcomes in the model. For instance, different regimes often produce different qualitative relationships between output and default, or output and settlements: with strength through weakness, defaults usually start when output is below trend, while with strength through strength, they typically begin when output is above trend. This points to the fact that changes in bargaining power have now become a primary de-

[4] The specific numerical value for the regime biased towards the debtor is 0.7. For the regime biased towards the debtor, this number is 0.01. These were chosen to display the full range of outcomes from this model.

terminant of the decision to default in the model, with the country more likely to default when it is in a strong bargaining position. Of the regimes considered, only the i.i.d. regime would not produce delay in equilibrium for calibrated income data (and if the procedure is biased towards the debtor would not show any default).

The results for the above regimes suggest that the model is a promising as regards its potential to produce delays in debt renegotiation. To assess its quantitative potential, we calibrate a bargaining power regime to some moments of the output data. To do so, we first reduce the number of parameters that describe the stochastic process. We assume that for states in which the debtor proposes, the probability that the debtor proposes in the next period can take only one of three values, depending on whether output is far below trend, near trend or far above trend. We also assume that the choice of subsequent proposer in states that follow a creditor's proposal is symmetric to that for the debtor; that is, the probability the creditor proposes repeatedly if current output is low is equivalent to the probability of a repeat choice of the debtor as proposer when output is high. We then estimate these three parameters by simulating and matching three moments from the data. These moments pertain to changes in income at the onset of default and the average distance of the economy relative to trend throughout default. We list these in Table 13.

The three parameter values we calibrate to correspond to the probability the debtor repeats as proposer if income income in the current period is low, π_l, moderate, π_m or high π_h. The values we find are $\pi_l = 0.91, \pi_m = 0.97$ and $\pi_h = 0.66$, which means that we are calibrating to a regime that can fit into the "strength through weakness" category, albeit one that is dissimilar to our example regime in many respects. It is also a highly persistent regime.

6.2 Examples of Delay

Next we provide a pair of examples of the type of outcomes traditionally associated with delay: one in which a persistent fall in output generates a lengthy renegotiation and one in which persistent changes in bargaining power generate a long stay in default.

For the first we consider a process by which the proposer is chosen i.i.d. In this process defaults occur when there are negative income shocks in the model and delays occur when income remains low for multiple periods. Note that in such a case the initial poor income shock does not necessarily lead to an immediate default. If the current debt level is low, the debtor may respond to a large shock by initially borrowing to smooth consumption. If income doesn't immediately recover, then the debtor defaults in the next period and may spend many periods in default. Such borrowing under the specter of default occurs at a large premium. It satisfies zero profits because a creditor holding debt with a high face value eventually obtains a significant payment.

In the second case, we consider a persistent process for the choice of the proposer. With this second process, at any given period, the values of the bargaining game are tilted towards whoever the current proposer is. Suppose the current proposer is a creditor. In such cases inside default, renegotiation is likely to involve small haircuts. Because of such a threat of renegotiation, outside default debtors can accumulate large amounts of debt. The threat of a default biased towards the creditor acts as a source of commitment, that allow debt to be accumulated. Should there be a shift in the current choice of proposer, a debtor would find it difficult to continue borrowing because he can discharge debt with very little payment in default. In addition, once the debtor enters default, he would find it difficult to obtain new borrowing as part of a settlement for the same reason.

The first scenario leads to default, the second leads to delay during default. Thus with a persistent process, defaults and long delays occur when there is a shift in the choice of proposer. Debt accumulates in states where the creditor proposes and is defaulted on when the debtor suddenly becomes the proposer. The debtor frequently stays in default (with a large stock of debt) until the creditor is chosen as the proposer.

For both examples we attach three pictures that pertain to one time path for default. We show the behavior of income, debt (in face value) and the actual recorded spread. Income includes the realized income plus a small output penalty due to default. The spread outside of (or exiting) default is just the inverse of the bond price The spread during (or entering) default is priced in secondary markets from the function W. pictures are attached as Figures 4-9.

We can highlight some general observations from these figures. In the i.i.d. example (Figure 4), the level of income is the key factor that determines when a default and when a settlement occur. Income must rebound to its value two periods prior to the initial default to reach settlement. This is because the negative shock which lead the default occurred in the period prior to default. As an immediate response to this shock the country ran up the face value of its debt after the shock and defaulted in the subsequent period. In the persistent case, Figure(7) default happened immediately with a bad shock, which was to bargaining power not to the endowment. The reason for this asymmetry is that income actually increased with the onset of default.

Next we can turn to prices. Note that the spreads in this model outside default are typically low. They increase sharply when a country defaults and fall in periods such that the country settles. In general the median of the spread over large time periods (without much default) is quite low. However the mean can be quite high, even if the average haircut in default is quite small. This is because a few very large spreads can occur in periods where the expected haircut approaches 1 even if such periods happen irregularly.

Finally we can talk a little bit about the face value and the market value of the debt (since the market value is just the face value multiplied by the spread). Consider first the i.i.d case. After the initial shock, the country increases the face value of the debt considerably (and the market value by a smaller amount.) The country then remains in default with this higher value of debt. In the period in which the renegotiations end, the market value of the debt expands considerably due to a fall in the bond price. However immediately after default, as part of the settlement debt returns to its pre-default level in the period in both face and market value. Because income is lower in the period after default then it was in the period before default, the debt to gdp ratio has actually risen as a result of the default. A similar story is true of the persistent case where the face value of debt falls with the resolution of default. However, the failure of income to recover to its initial level that the debt to gdp ratio (at market value) again increases as a result of this default. For the persistent case this is a common result as countries the period after default tend to have higher debt to gdp ratios than prior to entering default.

6.3 Results

In this section, we report key moments from these experiments. We group the results in this section around the six previous facts and present them in Tables 13 through 15. First some general observations are in order. The i.i.d regimes deserve a bit of special attention. With these regimes little delay is possible and with i.i.d regimes biased towards the debtor, default doesn't occur. For our income process, these two regimes lead to similar outcomes as does the model in Yue (2007). Thus a minimal test of our model is to compare the results of the calibrated model to the i.i.d examples.

Next we turn to a direct examination of the six facts previously described and assess our theory's performance in accounting for these facts. As we highlight, the model performs well on five out of six of these facts.

Fact 1: *sovereign defaults are time consuming to resolve*

We reproduce the data necessary to assess Facts 1-4 in Table 14. The mean number for delay lies in the first column. With the exception of the two iid regimes, the model performs well. In particular the benchmark can account for 66% of the delays present in the data.

Concurrent to fact one, we can report on the default probability, which is in column three of Table 14. Default probabilities in the model are a little bit lower than in our sample, but it is important to remember that our sample is conditioned on have experienced a default. Over longer time horizons, Arellano (2007) and Tomz and Wright (2007) have documented an average two per-cent probability of default per year which is in line with our model.

Fact 2: *creditor losses (or* haircuts*) are substantial*

We can check this fact from the second column in Table 14. This is the one fact the benchmark model has trouble matching. The reason for this is that large haircuts are a rare occurrence under this regime. Most defaults reach resolution when income is high and the creditor is the proposer. However since the degree of persistence in this case is relatively high, such defaults tend to be settled with small haircuts. Some defaults are settled when incomes are low and the creditor's bargaining advantage is very mild. These defaults tend to be associated with larger haircuts. But these defaults are far fewer in number.

Fact 3: *longer defaults are associated with larger haircuts*

This can be tested from examining the data in column 4 in Table 14. The theory consistently reproduces this correlation across a broad level of regimes.

Fact 4: *larger output declines in the year of default are associated with modestly longer defaults and larger haircuts*,

This can be tested by examining columns 5 and 6 in Table 14. The benchmark calibration performs relatively well along this dimension. When delays are driven by changes in bargaining regimes the stochastic process on bargaining regimes is more important for the timing of default than the stochastic process on income. In the calibrated case, the two are correlated.

Fact 5: *defaults are somewhat more likely to occur when output is below trend, and settlements tend to occur when output has returned to trend,*

We document the performance of the model along this dimension in Table 15. The model performs well on these measures across regimes. In assessing the performance of the benchmark calibration, we are somewhat aided by our calibration targets at the onset of default. On average, countries who default are above trend the period before the crisis and below trend in the period they default. In the data and the model these proportions are bounded away from one. As countries come to resolution they tend to approach trend, reaching trend only after the crisis ends and the output penalty disappears. The one counterexample for this behavior is a strength through strength regime, for which a large amount of settlements happen at low income values where creditors possess a large amount of bargaining power.

Fact 6: *default resolution is associated with increased country indebtedness.*

As shown in Table 16, this is also a widely held property of the model. Frequently countries respond to poor shocks by immediately expanding debt. Plus since income is below trend there are strong motives when debtors to include new borrowing in settlements.

Evidence for Evolving Bargaining Power

Our analysis suggests a strong mechanism that can account far a large portion of the excessive defaults experienced in the data. In terms of calibration this requires bargaining regimes that are somewhat persistent but do occasionally change. In this section we present evidence that sovereign debt since 1970's has been negotiated under such regimes.

Note, that for a time path of realized bargaining power, our model makes predictions about when countries default and settle. Suppose most of the nations of the world, borrow and lend under a similar regime. The presence of defaults over any time period require an initial realization of bargaining power that tilts the international regime towards the creditor. This is necessary for the country to be able to borrow more than trivial amounts outside of default, which is a precondition to actually defaulting. When the common international regime suddenly (or gradually) shifts its tilt to favor the debtor, large numbers of countries simultaneously default. A large proportion of those countries remain in default until the regime shifts back towards the creditor. Consequently if the international regime shifts from favoring the creditor to favoring the debtor and then back to favoring the creditor, we should expect the proportion of borrowers in default to start small, increase sharply en masse, and then decrease again as the regime shifts.

The historical experience from the 1970's through the 2000's suggests just this experience. Between 1955and 1970, there was little sovereign debt as creditors had little recourse to any court system and the emphasis on repayment in international financial markets or through the court system was very low. In the 1960's and early to mid 1970's however the emphasis in the court system began to grow. This was due to the erosion of the "absolute" view of sovereign immunity and movement to the "restrictive view" of sovereign immunity. The latter allows lawsuits against sovereigns for commercial activity. Europe codified this in 1972 with the European Convention on State Immunity and Additional Protocols, which was took force in 1976. US passed the International Sovereign Immunity Act in 1976. This represents a (gradual) shift in bargaining towards the creditors. As our theory predicts, the 1970's lead to an increase in sovereign lending. Contrary to our theory, default did increase at the end of the 1970's. However this was due to the paucity of debt entering this period (primarily in terms of amount borrowed, not number of borrowers) and default rates throughout the 1970's were significantly lower than their historical averages.

In the early 1980's, bargaining power shifted towards the debtors as international banks sought long term relationships with potential sovereigns in order to supply banking services to firms and residents of sovereign borrowers. These creditors cared a great deal access about access to local markets and were willing to sacrifice in renegotiation in order to support their other enterprizes. They were also willing to lend in arrears and frequently joined by the IMF in this regard. Concurrent to this shift, the percentage of countries in default exploded, reaching a post-WWI high in 1992.

Having lost large sums in Latin America in the 1980's, commercial banks retreated from the sovereign debt market and they were replaced in the 1990's by bondholders, who had no compulsion suing sovereign nations in US and UK courts, frequently leading to changes to US and UK laws in the process. Over time, bondholders earned the right to seize assets of sovereign countries, most famously in Eliot and Associates, 1997. The growth and strength of bondholders is traditionally thought of as shift in bargaining regime towards the creditors. This corresponds with a sharp fall in the proportion of borrowers in default in the mid 1990's. It should be pointed out that the historical correlation between debt in bonds and low proportions of borrowers in defaults runs against the predictions of most models of sovereign debt, however it is consistent with this model. Thus the increasing proportion of countries in default in the 1980's and the decreasing proportion in the 1990's is consistent with this model, but not many others.

7. CONCLUSION AND FUTURE WORK

We have presented a new database of sovereign defaults, the largest of its type that we are aware of. This database allowed us to describe the outcomes of sovereign defaults that had previously never been studied. We have demonstrated six facts about these outcomes. First we showed that defaults are frequently protracted. They last on average over seven years. But not all defaults are heavily protracted. The distribution is heavily skewed and the mode of delay is less than a year. Second, haircuts are severe, on average, 44% of the original debt. Third, somewhat surprisingly, countries end default more indebted than they had started. The average Debt to GDP ratio for a country just prior to default is 52%. After they settle, the average Debt to GDP ratio is 72%. Fourth, there is a strong correlation between haircuts and the length of default. The correlation coefficient between these variables is about two thirds. Fifth there is a modest negative relationship between economic conditions around the start of default and the length of the delay or the size of the haircut. These correlation coefficients are about one fifth. Sixth, the relationship between output and default is modest. Defaults happen when output is below trend, but only just. Resolutions occur when output has returned to trend.

We have in addition presented a theory where commitment problems cause both sovereign defaults and delays in renegotiation. We have calibrated the model so that it can be compared to the data. The theory is consistent with five of the six facts above, missing only on the average size of the haircut. In addition, we can produce default rates that are closer to the data than what more established models show and large debt to GDP ratios, usually a stumbling block for this type of model.

The theory has the added benefit that changes in bargaining regimes can be the driving force leading countries into and out of default. In practice, countries that are contemporaries to each other tend to renegotiate under similar institutions. Because of this, our theory can account for one of the more vexing issues in the sovereign debt literature: the the herding into default witnessed in Latin America in the 1980's. To see this, note that many policies affect bargaining power in a manner that it is easy to attribute a winner to. This is true of many actual policies in the 1980's and 1990's. Given the attributed winner, these policy changes had the results the theory predicts. Those that favored the debtor, paradoxically lead to more countries in default and those that favored creditors lead to more countries exiting default for the international market. From this experience, the logical policy advice is that policy makers should be careful of reforms that erode the ability of sovereign nations to commit to repaying its debt. This is true even if countries have recently shown a lack of ability to repay its debt.

We intend to pursue three extensions of this project in future work. First the model makes strong predictions about the relationship between sovereign debt spreads and the timing of default. There is much here that deserves testing. Default occurs when spreads rise and ends when spreads return to pre-crisis levels. In addition, the model predicts that spreads in default that are sixty-five times greater than spreads inside default. The relationship between the timing of default and the bond price is the central mechanism of the paper. Detailed collection of spreads on primary markets outside default and on secondary markets inside default should be collected to see if they match predictions.

Second, there are additional policy prescriptions that don't map into bargaining power. In particular bailouts are often discussed as a means of helping countries in sovereign default. This theory can help shed light on the welfare effects of bailouts. It is not obvious ex-ante what these welfare effects are. This is in part because the effect of delay on ex ante welfare is itself ambiguous, depending on whether it discourages all default or just excessive default. When studying bailouts there are two issues worth concern, the timing and the level of the bailout. Consider a deep pocketed international institution. It would be worthwhile to discover which lengths and degrees of bailout, if any, would raise ex ante utility by more than the cost of such a bailout.

Third and finally, we have calibrated the model on a quarterly frequency. Although this is standard when examining the timing of investment decisions in macroeconomics, it is arguably too long a time horizon when thinking about the frequency with which parties may make proposals in bargaining. In shortening the time horizon, it is also necessary to change the level of persistence of the income and proposer process, and also to vary the set of assets available to the country; otherwise, we might end up with a model calibrated to a monthly frequency in which the country can only issue thirty-day treasury bills. Adding more assets, however, expands the dimension of the state vector for the model, and hence requires greater computational power.

REFERENCES

[1] Aguiar, Mark and Gita Gopinath. 2006. "Defaultable Debt, Interest Rates and the Current Account." *Journal of International Economics*, 69:1, pp. 64-83.

[2] Aliprantis, Charalambos D. and Kim C. Border. 2006. *Infinite Dimensional Analysis: A Hitchhikers Guide*. Berlin: Springer-Verlag, 3rd Edition.

[3] Arellano, Cristina. 2006. "Default Risk and Income Fluctuations in Emerging Economies." *University of Minnesota Working Paper*.

[4] Arteta, Carlos and Galina Hale. 2008. "Sovereign debt crises and credit to the private sector." *Journal of International Economics*, 74:1, pp. 53-69.

[5] Baron, David P. and John A. Ferejohn. 1989. "Bargaining in Legislatures." *The American Political Science Review*, 83:4, pp. 1181-206.

[6] Beers, David T. and John Chambers. 2006. "Default Study: Sovereign Defaults At 26-Year Low, To Show Little Change In 2007". *Standard & Poor's CreditWeek*, September 18.

[7] Bi, Ran. 2008. "“Beneficial” Delays in Debt Restructuring Negotiations." *IMF Working Paper*, 08:38.

[8] Binmore, Ken, Martin J. Osborne, and Ariel Rubinstein. 1992. “Noncooperative models of bargaining,” in: R.J. Aumann & S. Hart (ed.), *Handbook of Game Theory with Economic Applications*, volume 1, chapter 7, pages 179-225 Elsevier.

[9] Bulow, Jeremy and Kenneth Rogoff. 1989. "A Constant Recontracting Model of Sovereign Debt." *The Journal of Political Economy*, 97:1, pp. 155-78.

[10] Bolton, Patrick and David S. Scharfstein. 1996. "Optimal Debt Structure and the Number of Creditors." *The Journal of Political Economy*, 104:1, pp. 1-25.

[11] Chatterjee, Satyajit, Dean Corbae, Makoto Nakajima, and Jose-Victor Rios-Rull. 2007. "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default." *Econometrica*, 75:6, pp. 1525-89.

[12] Cline, William R. 1995. *International Debt Re-examined*. Washington, DC: Institute for International Economics.

[13] Cripps, Martin W. 1998. "Markov bargaining games." *Journal of Economic Dynamics and Control*, 22:3, pp. 341-55.

[14] Diermeier, Daniel, Hülya Eraslan, and Antonio Merlo. 2003. "A Structural Model of Government Formation." *Econometrica*, 71:1, pp. 27-70.

[15] Eaton, Jonathan and Mark Gersovitz. 1981. "Debt with potential repudiation: Theoretical and empirical analysis." *Review of Economic Studies*, 48:2, pp. 289-309.

[16] Eraslan, Hülya. 2008. "Corporate Bankruptcy Reorganizations: Estimates From a Bargaining Model," *International Economic Review*, forthcoming May, 2008.

[17] Global Committee of Argentina Bondholders. 2004. *Investor Roadshow Presentation*. Global Committee of Argentina Bondholders: New York.

[18] Institute for International Finance (2001). *Survey of Debt Restructuring by Private Creditors*. Institute for International Finance: Washington, D.C.

[19] Kovrijnykh, Natalia and Balázs Szentes. 2007. "Equilibrium Default Cycles." *Journal of Political Economy*, 115:3, pp. 403-46.

[20] Mendoza, Enrique G. and Vivian Z. Yue. 2007. "Solving the Country Risk - Business Cycle Disconnect: Endogenous Output Collapse in a Model of Sovereign Default." *Unpublished Paper, University of Maryland*.

[21] Merlo, Antonio and Charles Wilson. 1995. "A Stochastic Model of Sequential Bargaining with Complete Information." *Econometrica*, 63:2, pp. 371-99.

[22] Merlo, Antonio and Charles Wilson. 1997. "Efficient delays in a stochastic model of bargaining." *Economic Theory*, 11:1, pp. 39-55.

[23] Pitchford, Rohan and Mark L. J. Wright. 2008. "Restructuring the Sovereign Debt Restructuring Mechanism." *UCLA Working Paper.*

[24] Ravn, Morten O. and Harald Uhlig. 2002. "On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations." *Review of Economics and Statistics*, 84:2, pp. 371-76.

[25] Rubinstein, Ariel 1982. "Perfect Equilibrium in a Bargaining Model." *Econometrica*, 50:1, pp. 97-110.

[26] Shaked, Avner. 1994. "Opting Out: Bazaars versus "High Tech" Markets." *Investigaciones Económicas*, 18:3, pp. 421-32.

[27] Sturzenegger, Federico and Jeromin Zettelmeyer. "Haircuts: Estimating investor losses in sovereign debt restructurings, 1998-2005." *Journal of International Money and Finance*, In Press, Corrected Proof.

[28] Sturzenegger, Federico and Jeromin Zettelmeyer. 2005. "Haircuts: Estimating Investor Losses in Sovereign Debt Restructurings, 1998–2005." *IMF Working Paper*, 05:137.

[29] Tomz, Michael and Mark L. J. Wright. 2007. "Do Countries Default in "Bad Times"?" *Journal of the European Economic Association*, 5:2-3, pp. 352-60.

[30] Trebesch, Christoph. 2008. "Delays in Sovereign Debt Restructuring: Should We Really Blame the Creditors?" *Unpublished Paper, Free University of Berlin.*

[31] World Bank. various. *Global Development Finance*. World Bank: Washington, D.C.

[32] Yue, Vivian Z. 2006. "Sovereign Default and Debt Renegotiation." *New York University Working Paper*.

Table 1: Delays and Haircuts

	Mean	correlation with		
		Δy	$\Delta^2 y$	Debt/GDP
Delay 1	7.5 years	-0.26	-0.21	0.03
Delay 2	8.1 years	-0.26	-0.21	0.03
Haircut	44%	-0.25	-0.23	0.02

Figure 2: The Relationship Between Delays and Haircuts

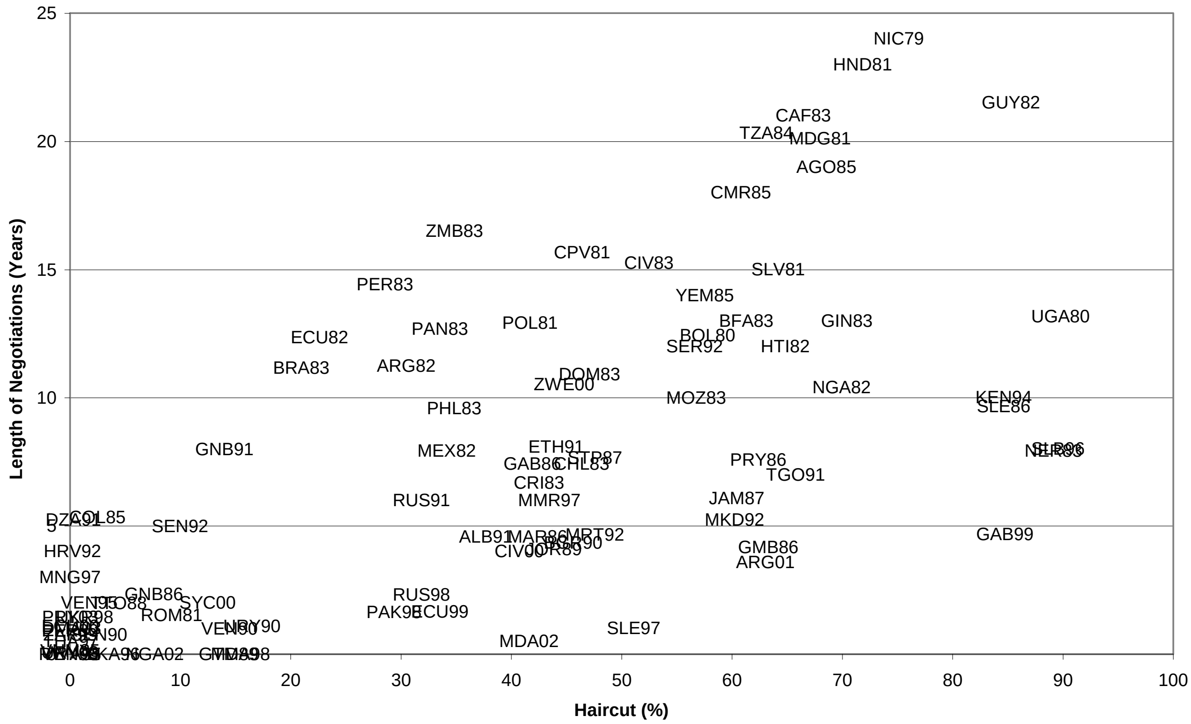


Table 3: Output and Debt Levels Around Default

	trend dev. Y (%)	% obs below trend	average debt/gdp
in default	-0.4	53.8	87.3
out of default	0.4	43.0	50.9
year before default	1.0	38.9	51.9
year of default	-1.3	64.0	58.0
year of settlement	-0.2	51.1	72.6
year after settlement	0.1	47.9	72.0

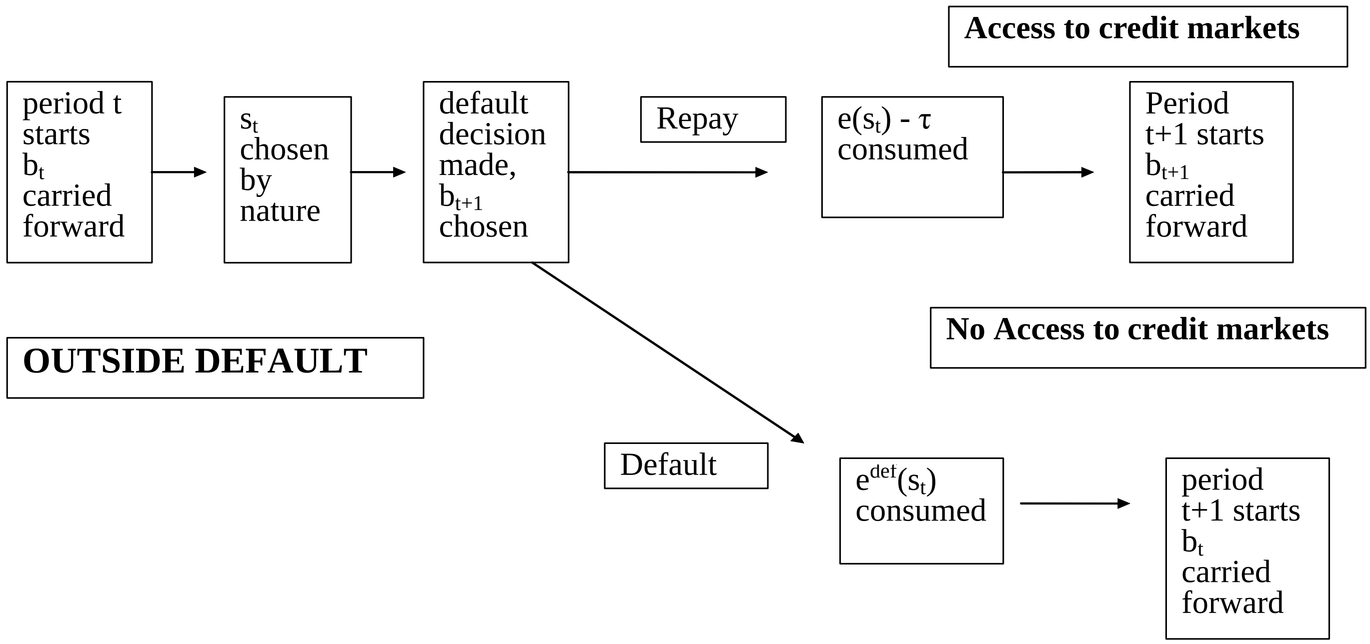


Figure 4: Timeline of Decisions Outside Default

Figure 5: Timeline of Decision Inside Default

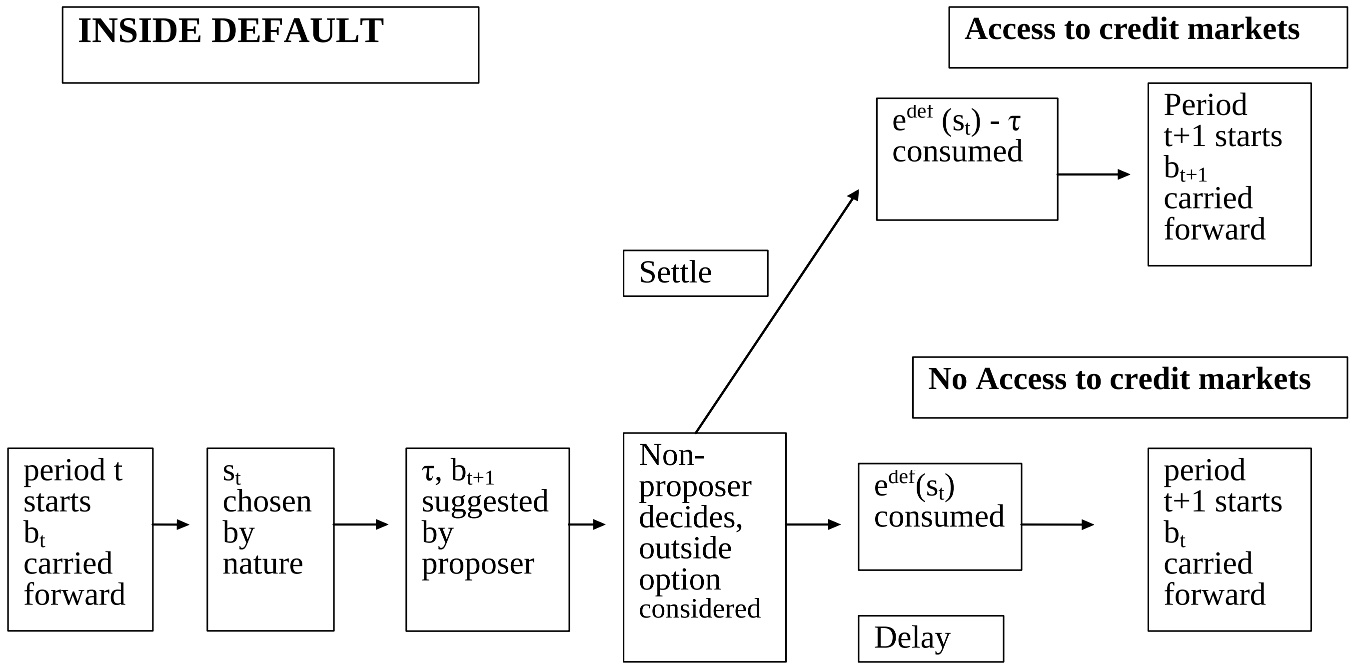


Figure 6: Example Income Process 1

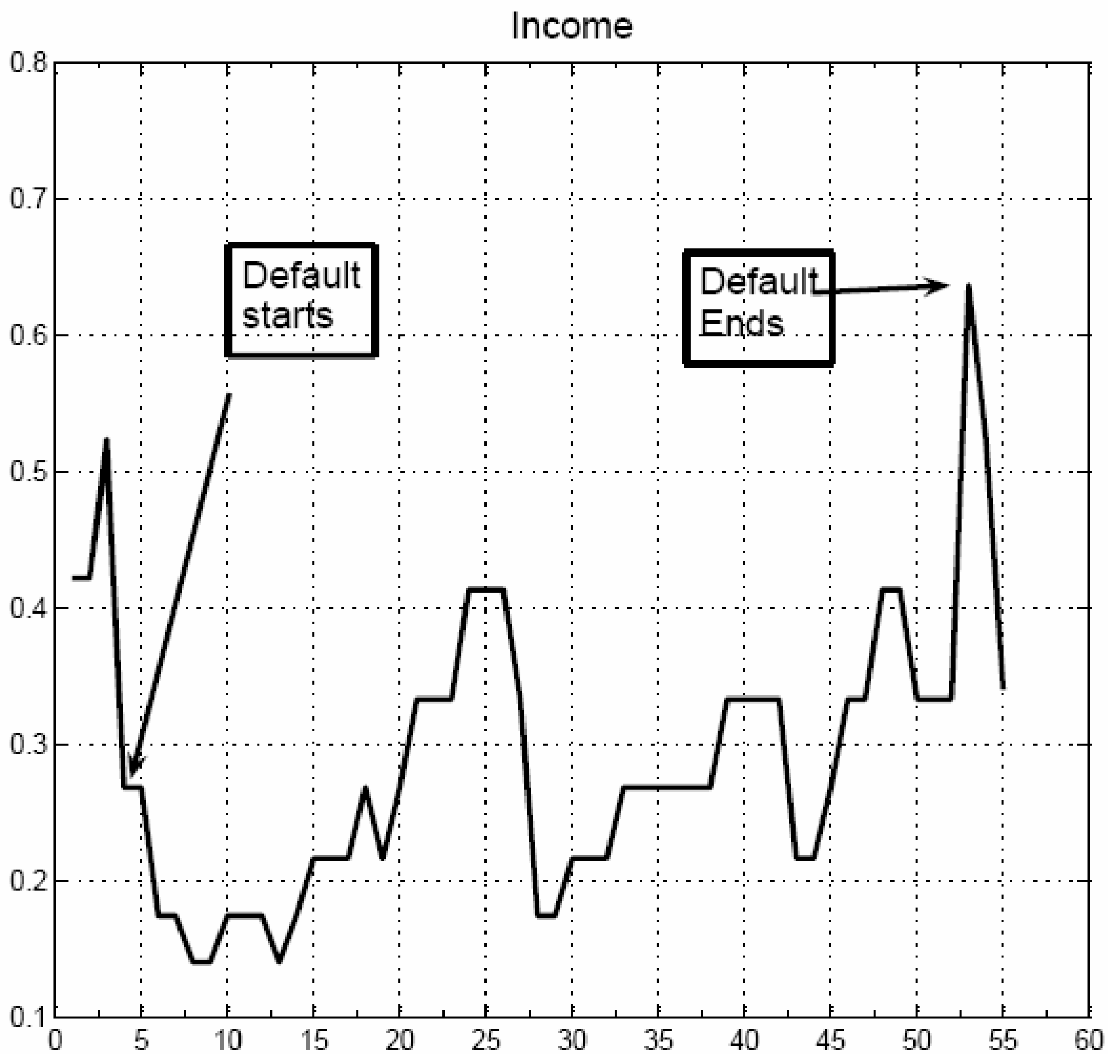


Figure 7: Example Spreads 1

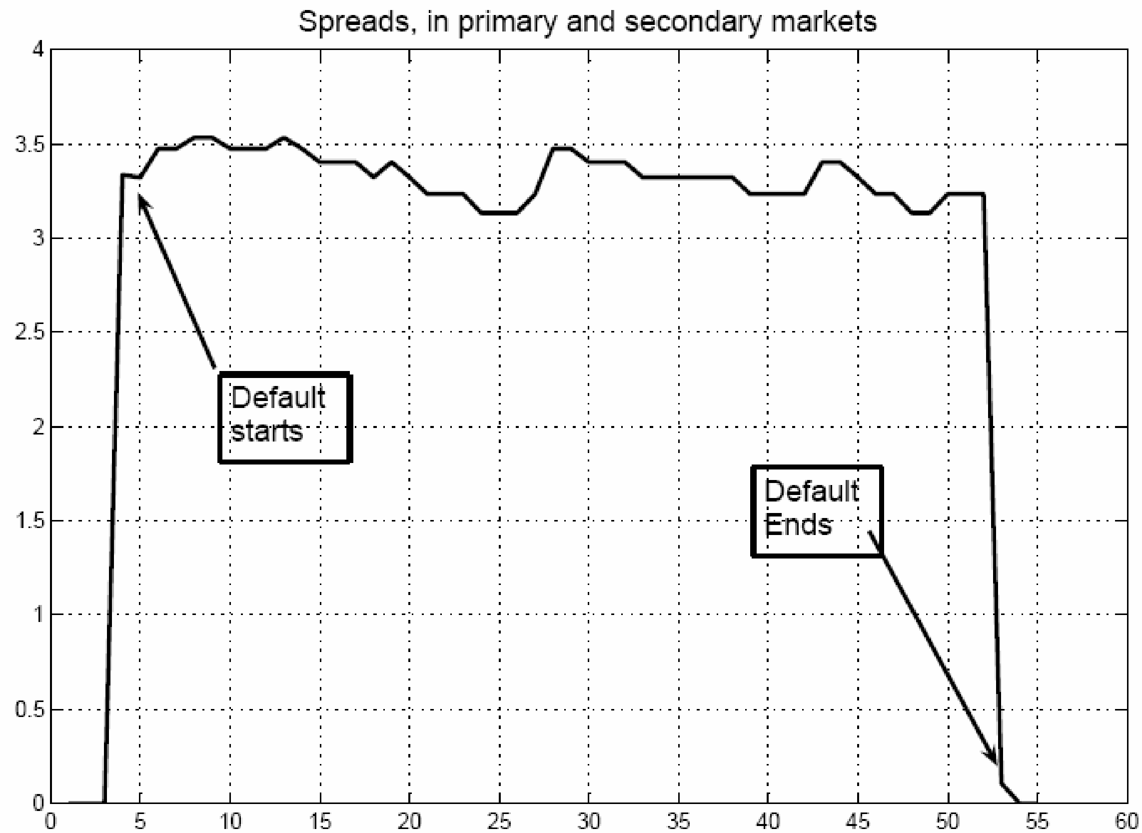


Figure 8: Debt in Default 1

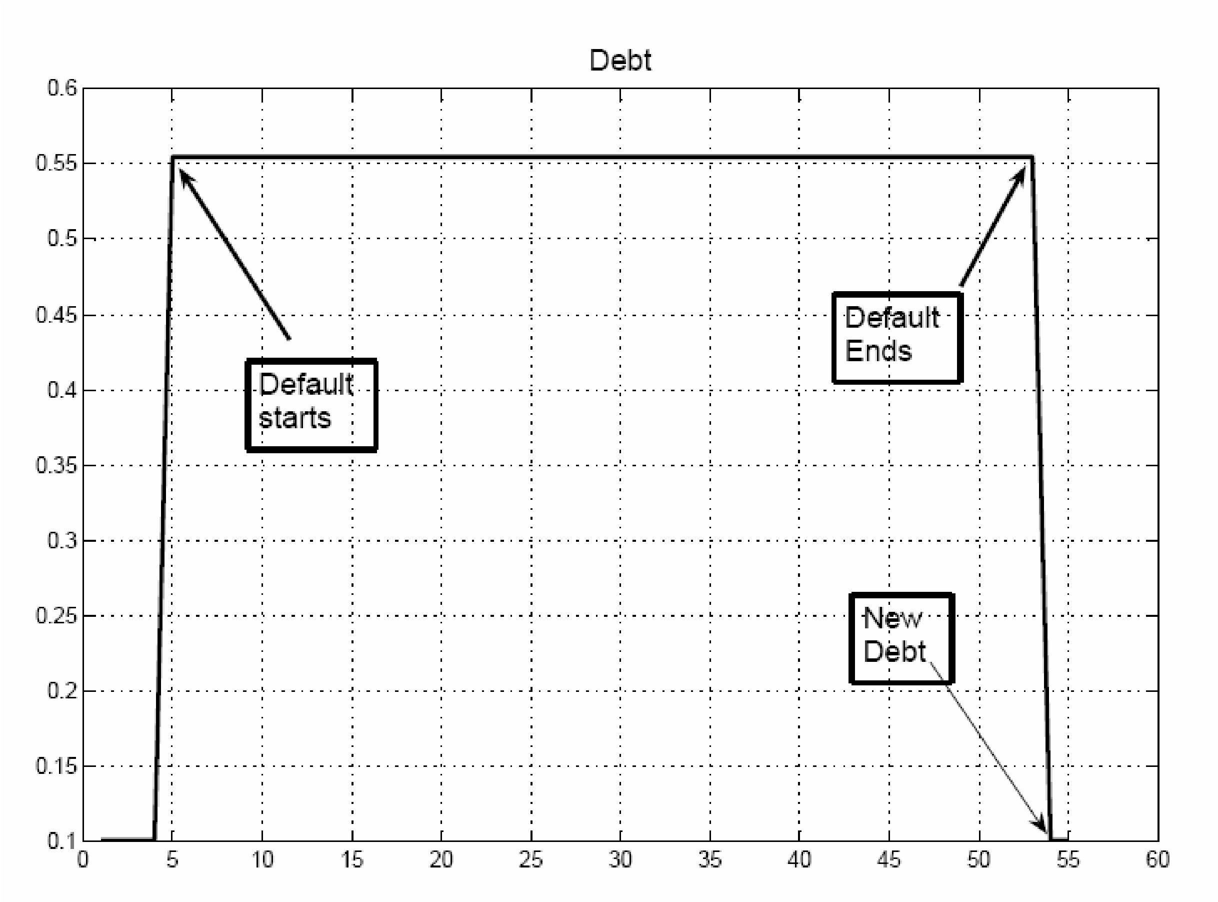


Figure 9: Example Income Process 2

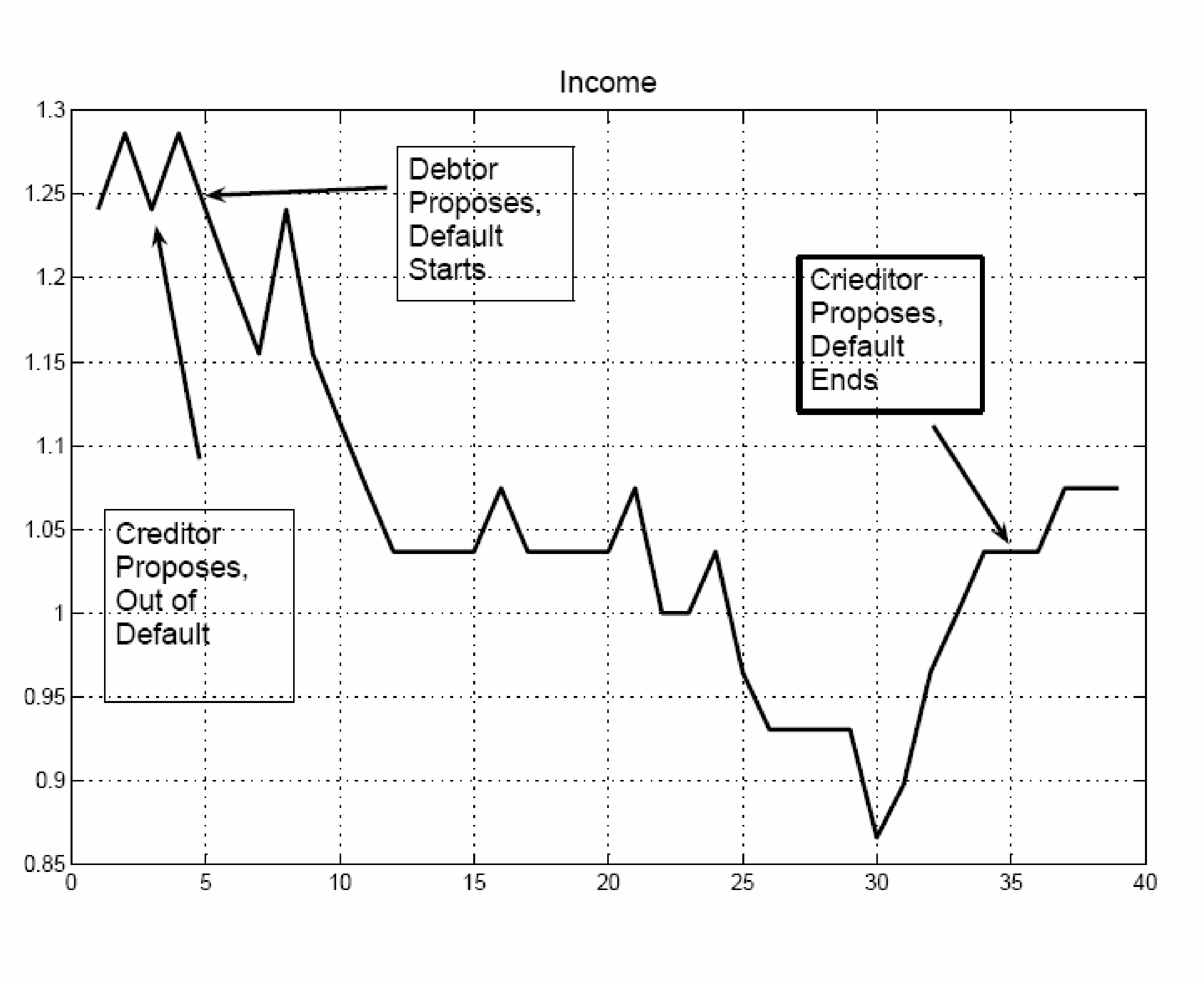
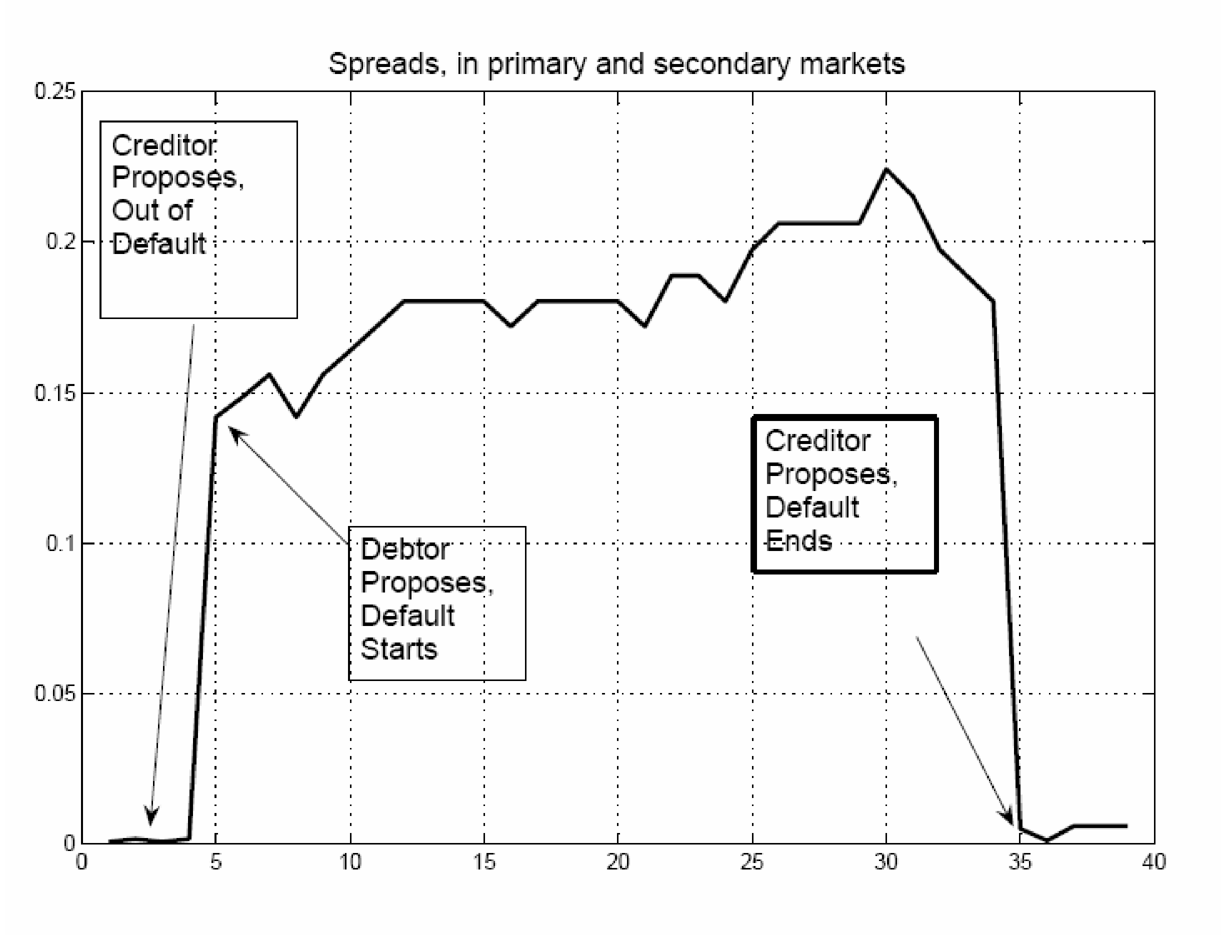


Figure 10: Example Spreads



2

Figure 11: Example Spreads 2

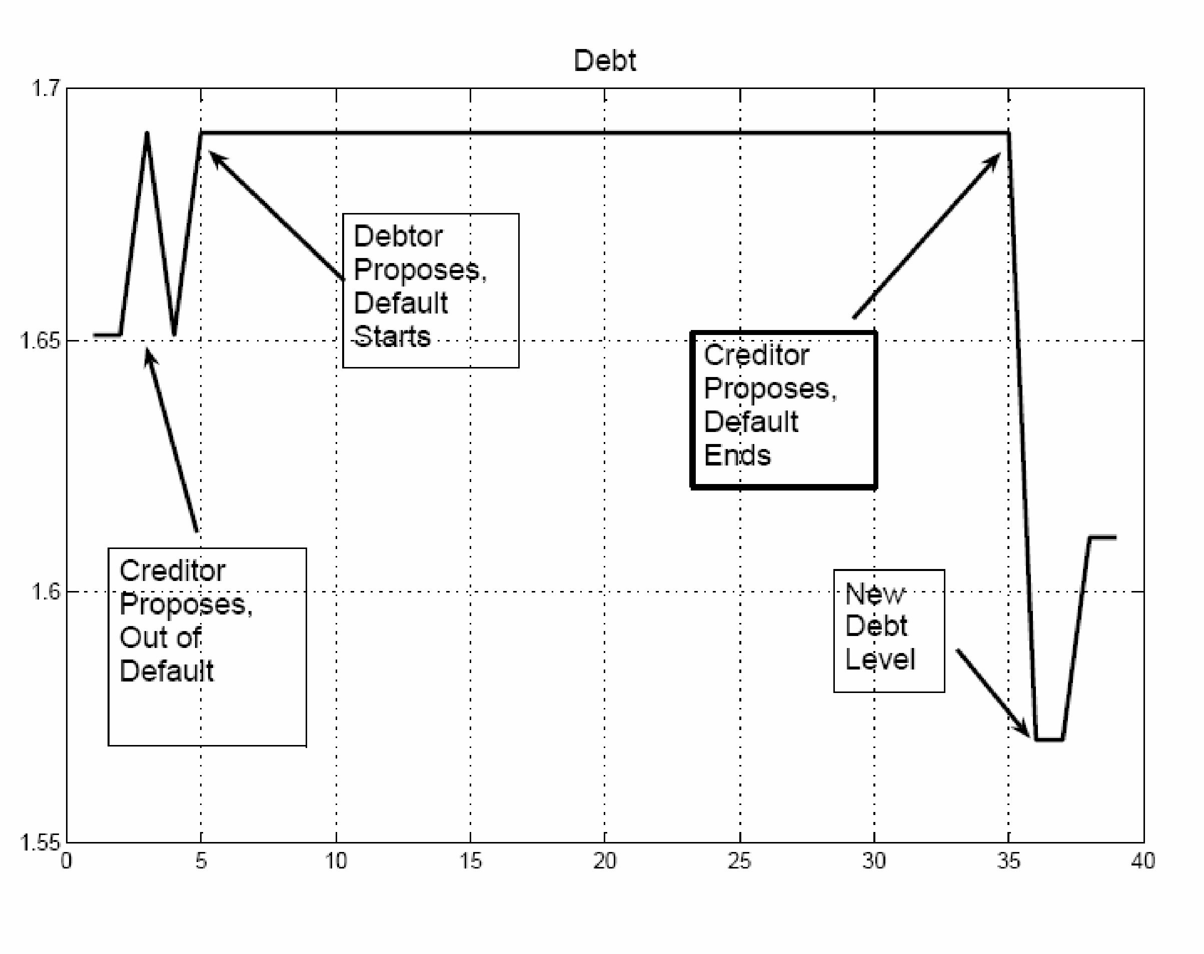


Table 12: Parameter Values for Calibration

Name	Meaning	Value
β	Discount factor	0.945
r^w	World Interest rate	1.01
γ	CRRA	2
μ	Mean	0.72
ε	Std Dev	0.095
λ	Ouput loss	0.02

Table 13: Calibration Targets

Target	Data	Model Outcome
mean(income, default)	-0.0042	-0.0036
mean(Δ income, period before default)	-0.012	-0.012
mean(Δ income, starting period	-0.024	-0.021

Table 14: Numerical Results for Delays and Haircuts

Regime	Mean(t)	Mean (h)	Default Prob	Corr(t,h)	Corr(t,e)	Corr(t,h)
i.i.d debt	-	-	-	-	-	-
i.i.d cred.	1.01	0.39	0.01	18	.00	-0.02
strength through strength	9.28	12.3	0.05	0.51	0.03	0.02
strength through weakness	18.21	13.9	0.03	0.52	0.06	0.15
persistent	78.3	36.1	0.01	42.5	0.05	0.20
benchmark	21.5	0.08	0.03	0.74	-0.20	-0.33
data	0.04	32.4	0.44	0.66	-0.19	-0.28

t=delay

h=haircut

e=change in output over two periods prior to default

Table 15: Numerical Results on the Proportion of Countries Below Trend

Regime	Period Before	Entering	Exiting	Period After
i.i.d debt	-	-	-	-
i.i.d cred.	0.94	0.98	0.98	0.82
strong	0.21	0.55	0.61	0.58
weak	0.70	0.87	0.69	0.19
pers	0.42	0.67	0.64	0.40
bench	0.29	0.84	0.75	0.19
data	0.40	0.64	0.52	0.49

Table 16: Numerical Results on Debt To GDP

Regime	Period Before	Entering	Exiting	Period After	On Average
i.i.d debt	-	-	-	-	0.22
i.i.d cred.	1.74	1.72	1.72	1.68	1.64
strong	0.80	0.83	0.83	0.81	0.80
weak	0.82	0.84	0.84	0.83	0.83
pers	0.92	0.94	0.98	0.98	1.35
bench	1.20	1.23	1.25	1.26	1.29
data	0.50	0.57	0.70	0.70	0.65

A1. DATA APPENDIX

In this appendix, we tabulate our data on delays and haircuts, and study the relationship betwene our estimates of haircuts and those computed by other authors. The data are presented first in Table 17, for all ninety defaults and settlements.

Table 18 then presents the correlations between our measures of haircuts, and those computed by other authors for smaller samples of countries. As shown in the Table, the correlation with the World Bank and Cline estimates is around 0.9, which presumably follows from the similar sources of data. The correlation with the Sturzenegger and Zettelmeyer preferred estimate (calculated as a debt value weighted average over the estimates for all instruments in a restructuring) is also 0.86. Interestingly, the correlations with the market estimates of Sturzenegger and Zettelmeyer, and with the estimates produced by the Global Committee of Argentine Bondholders, are the smallest.

Table 18 also presents results for the relationship between delays and the different measures of haircuts. As shown in the table, the range of estimates brackets the one produced for the large sample (0.66). The most reliable estimates, produced by Sturzenegger and Zettelmeyer, have the highest correlation with delays at 0.88.

Table 17A: Data on Delays and Haircuts

			Length (Years)			Haircuts (%)					
	Country	Default	Default	Default	Default	Authors	World Bank	Cline	Sturzenegger & Zettelmeyer (2005)		GCAB
Country	Code	Code	Start	End	Length	Estimate	(1993)	(1995)	Preferred	Market	(2004)
Albania	ALB	ALB91	1991	1995	4.6	38					
Algeria	DZA	DZA91	1991	1996	5.2	0					
Angola	AGO	AGO85	1985	2004	19.0	69					
Argentina	ARG	ARG82	1989	1990	11.2	30	32	29			35
Argentina	ARG	ARG01	2001	2005	3.6	63			55	63	63
Bolivia	BOL	BOL80	1980	1993	12.4	58	78				
Brazil	BRA	BRA83	1983	1994	11.2	21	18	28			35
Bulgaria	BGR	BGR90	1990	1994	4.3	46	44	50			50
Burkina Faso	BFA	BFA83	1983	1996	13.0	61					
Cameroon	CMR	CMR85	1985	2003	18.0	61					
Cape Verde	CPV	CPV81	1981	1996	15.7	46					
Central African Republic	CAF	CAF83	1983	2004	21.0	66					
Chile	CHL	CHL83	1983	1990	7.4	46					
Colombia	COL	COL85	1985	1991	5.3	2					
Costa Rica	CRI	CRI83	1983	1990	6.7	43	62	61			
Croatia	HRV	HRV92	1992	1996	4.0	0					
Dominica	DMA	DMA03	2003	2004	1.0	0					
Dominican Republic	DOM	DOM83	1983	1994	10.9	47	63	50			
Ecuador	ECU	ECU82	1982	1995	12.3	23		45			45
Ecuador	ECU	ECU99	1999	2000	1.7	34			27	60	
Ecuador	ECU	ECU00	2000	2001	1.1	0					40
El Salvador	SLV	SLV81	1981	1996	15.0	64					
Ethiopia	ETH	ETH91	1991	1999	8.1	44					
Gabon	GAB	GAB86	1986	1994	7.4	42					
Gabon	GAB	GAB99	1999	2004	4.7	85					
Gambia	GMB	GMB86	1986	1990	4.2	63					
Guatemala	GTM	GTM89	1989	1989	0.0	14					
Guinea	GNB	GNB86	1986	1988	2.3	8					
Guinea	GNB	GNB91	1991	1998	8.0	14					
Guinea-Bissau	GIN	GIN83	1983	1996	13.0	70					
Guyana	GUY	GUY82	1982	2004	21.5	85		86			
Haiti	HTI	HTI82	1982	1994	12.0	65					
Honduras	HND	HND81	1981	2004	23.0	72					
Ivory Coast	CIV	CIV83	1983	1998	15.2	52					
Ivory Coast	CIV	CIV00	2000	2004	4.0	41					
Jamaica	JAM	JAM87	1987	1993	6.1	60					
Jordan	JOR	JOR89	1989	1993	4.1	44	42	33			35
Kenya	KEN	KEN94	1994	2004	10.0	85					
Macedonia	MKD	MKD92	1992	1997	5.2	60					
Madagascar	MDG	MDG81	1981	2002	20.1	68					
Mauritania	MRT	MRT92	1992	1996	4.7	48					
Mexico	MEX	MEX82	1982	1990	7.9	34	35	30			35
Moldova	MDA	MDA98	1998	1998	0.0	15					
Moldova	MDA	MDA02	2002	2002	0.5	42					
Mongolia	MNG	MNG97	1997	2000	3.0	0					

Table 17B: Data on Delays and Haircuts (Continued)

			Length (Years)			Haircuts (%)					
	Country	Default	Default	Default	Default	Authors	World Bank	Cline	Sturzenegger & Zettelmeyer (2005)		GCAB
Country	Code	Code	Start	End	Length	Estimate	(1993)	(1995)	Preferred	Market	(2004)
Morocco	MAR	MAR86	1986	1990	4.6	42					
Mozambique	MOZ	MOZ83	1983	1992	10.0	57		58			
Myanmar	MMR	MMR97	1997	2003	6.0	43					
Nicaragua	NIC	NIC79	1979	2003	24.0	75					
Niger	NER	NER83	1983	1991	7.9	89		82			
Nigeria	NGA	NGA82	1982	1992	10.4	70	80	49			
Nigeria	NGA	NGA02	2002	2002	0.0	8					
Pakistan	PAK	PAK98	1998	1999	1.6	29			31	30	
Panama	PAN	PAN83	1983	1996	12.7	34					45
Paraguay	PRY	PRY86	1986	1993	7.6	62					
Paraguay	PRY	PRY03	2003	2004	1.4	0					
Peru	PER	PER80	1980	1980	0.9	0					
Peru	PER	PER83	1983	1997	14.4	29					45
Philippines	PHL	PHL83	1983	1992	9.6	35	44	36			
Poland	POL	POL81	1981	1994	12.9	42	58	45			45
Romania	ROM	ROM81	1981	1983	1.5	9					
Russia	RUS	RUS91	1991	1997	6.0	32					
Russia	RUS	RUS98	1998	2000	2.3	32			53	65	38
Rwanda	RWA	RWA95	1995	1995	0.0	0					
Sao Tome and Principe	STP	STP87	1987	1994	7.7	48					
Senegal	SEN	SEN90	1990	1990	0.7	3					
Senegal	SEN	SEN92	1992	1996	5.0	10					
Serbia and Montenegro	SER	SER92	1992	2004	12.0	57					
Seychelles	SYC	SYC00	2000	2002	2.0	12					
Sierra Leone	SLE	SLE86	1986	1995	9.7	85					
Sierra Leone	SLE	SLE97	1997	1998	1.0	51					
Solomon Islands	SLB	SLB96	1996	2004	8.0	90					
South Africa	ZAF	ZAF93	1993	1993	0.7	0					
Sri Lanka	LKA	LKA96	1996	1996	0.0	4					
Tanzania	TZA	TZA84	1984	2004	20.3	63					
Thailand	THA	THA97	1997	1998	0.5	0					
Togo	TGO	TGO91	1991	1997	7.0	66					
Trinidad and Tobago	TTO	TTO88	1988	1989	2.0	4					
Uganda	UGA	UGA80	1980	1993	13.2	90		76			
Ukraine	UKR	UKR98	1998	2000	1.4	1			18	28	
Uruguay	URY	URY90	1990	1991	1.1	16	41	31			
Uruguay	URY	URY03	2003	2003	0.0	0			16	29	
Venezuela	VEN	VEN90	1990	1990	1.0	14	23	20			30
Venezuela	VEN	VEN95	1995	1997	2.0	2					
Venezuela	VEN	VEN98	1998	1998	0.0	0					
Venezuela	VEN	VEN05	2005	2005	0.1	0					
Vietnam	VNM	VNM85	1985	1998	14.0	58					
Yemen	YEM	YEM85	1985	2001	16.5	35					
Zambia	ZMB	ZMB83	1983	1994	10.5	45					
Zimbabwe	ZWE	ZWE00	2000	2004	4.0	19					

Table 18: Comparison of Alternate Haircut Estimates

	World Bank (1993)	Cline (1995)	Sturzenegger and Zettelmeyer (2006) Preferred	Sturzenegger and Zettelmeyer (2006) Market	GCAB (2004)
no. obs.	13	17	6	6	17
Corr. with Authors'	0.87	0.90	0.86	0.77	0.50
Corr. with Delay	0.40	0.55	0.88	0.72	0.42