# International Capital Flows under Dispersed Information: Theory and Evidence[1]

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**Abstract**

We develop a new theory of international capital flows that is based on dispersed information. There is extensive evidence of information heterogeneity within and across countries, which has been critical for understanding asset prices. We analyze the implications for capital flows by introducing information dispersion into an open economy dynamic general equilibrium portfolio choice model. We show that each of the standard elements of portfolio allocation (changes in wealth, expected returns and risk) is affected by dispersed information. We emphasize two implications for both gross and net capital flows: (i) they become partially disconnected from observed macro fundamentals and (ii) they contain information about future macro fundamentals. These implications are confronted to data for industrialized countries.

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# 1  Introduction

Heterogeneity of information, both within and across countries, has been widely documented. Survey evidence shows that expectations about future macro fundamentals and asset prices differ widely across financial institutions and individuals. In addition there is evidence of information differences across countries. For example, it has been documented that local analysts have better quality information than foreign analysts and that agency problems are better monitored by locals.[1] There is abundant evidence that information dispersion is important for understanding asset prices. For example, the close link between exchange rates and cumulative order flow documented by Evans and Lyons (2002) suggests that exchange rates are driven largely by private information that is aggregated through order flow. The noisy rational expectations literature has shown that dispersed information can shed light on a wide variety of stylized facts about asset pricing.[2]

The main goal of the paper is to develop a general equilibrium theory of international capital flows under dispersed information. Like asset prices, dispersed information about expected returns can be expected to affect capital flows as well. If correct, this leads to an entirely new understanding of what drives both gross and net capital flows than based on existing common knowledge models. We emphasize two implications of dispersed information. First, it leads to a partial disconnect of capital flows from observed macro fundamentals. Second, capital flows contain information about future macro fundamentals. We confront these implications to data for industrialized countries.

The theory we develop integrates elements of noisy rational expectations (NRE) models commonly used in the finance literature into a full dynamic stochastic general equilibrium (DSGE) open-economy portfolio choice model. The two standard features of NRE models are private information about future fundamentals and unobserved portfolio shifts (noise) that prevents asset prices from revealing the private information. We integrate these elements into a two-country general equilibrium model where agents make decisions about portfolio allocation, physical

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[1] See respectively Bae, Stulz and Tan (2007) and Leuz, Lins and Warnock (2008).

[2] Albuquerque and Miao (2008) show that it can explain asset price momentum and reversal. Wang (1994) uses a model with dispersed information to explain the observed link between equity prices and trading volume. Bacchetta and van Wincoop (2006) show that the relationships between exchange rates, macro fundamentals and order flow can be explained in a model with dispersed information.

investment and saving. We avoid many special assumptions made in the NRE literature that facilitate the solution but do not connect well to the DSGE macroeconomics literature. Three assumptions are particularly unpalatable in the context of macroeconomic models: NRE models are entirely linear, adopt a riskfree asset that is in infinite supply and assume CARA preferences. The second assumption implies that standard NRE models are not truly general equilibrium models.

We analyze capital flows by adopting a portfolio perspective, deriving both capital inflows and outflows as a function of all the standard elements of portfolio allocation: changes in wealth (saving), changes in expected returns and changes in the risk-characteristics of assets. We show that information dispersion affects each of these components of capital flows. Changes in unobserved state variables (future fundamentals and unobserved portfolio shifts) affect asset prices, which affect saving, investment and equilibrium expected returns. They also lead to differences in expected returns across countries and time-varying second moments that affect portfolio risk.

The paper makes a methodological contribution as well by solving a model that introduces information dispersion into a full fledged DSGE setup with portfolio choice. Only recently has the literature begun to investigate DSGE open economy models with portfolio choice and to develop methods for solving them.[3] However, these models do not contain information dispersion. Standard methods for solving NRE models cannot be applied either because of the many special assumptions listed above. The solution we develop combines and extends methods for solving standard NRE models with recently developed local approximation methods for solving DSGE models with portfolio choice. Even though the combined presence of DSGE and NRE features makes the model quite rich, we are nonetheless able to obtain an analytical solution. This facilitates transparency of the results.

The paper is related to a small set of papers that have introduced NRE asset pricing features into open economy models. These include Albuquerque, Bauer and Schneider (2006), Bacchetta and van Wincoop (2004,2006), Brennan and Cao (1997), Gehrig (1993) and Veldkamp and van Nieuwerburgh (2007). These papers focus on a variety of issues, ranging from exchange rate puzzles to international portfolio home bias and the relationship between asset returns and portfolio flows. Together they show that information dispersion and information asymmetries can

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[3]See Devereux and Sutherland (2007), Evans and Hnatkovska (2007) and Tille and van Wincoop (2008).

tell us a lot about a wide range of stylized facts related to international asset prices and portfolio allocation. However, none of these papers have implications for *aggregate* capital inflows and outflows or even net capital flows. This is not just because the focus is on other questions but more fundamentally because these are not true general equilibrium models due to the presence of a riskfree asset with a constant return that is in infinite supply.

The paper is organized as follows. Section 2 describes the model. The solution method is discussed in section 3. Section 4 derives implications for asset prices, portfolio allocation and capital flows. This leads to three testable implications that are brought to the data in section 5. Section 6 concludes.

# 2   The Model

There are two countries, Home and Foreign, with a unit mass of atomistic agents in each country. Both countries produce the same good using labor and capital. The good can be used for consumption or investment, the latter entailing an adjustment cost. We adopt a standard overlapping generation setup. When young, agents earn labor income and make consumption and portfolio decisions. They can invest in claims on capital in both countries. While these are claims on aggregate capital rather than residual claims, we refer to them as Home and Foreign equity for convenience. During the second period of life, when old, agents consume the return on their investment.

## 2.1   Production, Investment and Assets

The consumption good is taken as the numeraire. It is produced in both countries using a constant returns to scale technology in labor and capital:

$$Y_{i,t} = A_{i,t} K_{i,t}^{1-\omega} N_{i,t}^{\omega} \quad i = H, F \tag{1}$$

where $H$ and $F$ denote the Home and Foreign country respectively. $Y_i$ is the output in country $i$, $A_i$ is a country-specific exogenous stochastic productivity term, $K_i$ is the capital input and $N_i$ the labor input that we normalize to unity. The log productivity follows an autoregressive process:

$$a_{i,t+1} = \rho a_{i,t} + \varepsilon_{i,t+1} \quad i = H, F$$

where $\varepsilon_{i,t+1}$ has a $N(0, \sigma_a^2)$ distribution and is uncorrelated across countries.

The dynamics of the capital stock reflects depreciation at a rate $\delta$ and investment $I_{i,t}$:

$$K_{i,t+1} = (1 - \delta) K_{i,t} + I_{i,t} \quad i = H, F \tag{2}$$

A share $\omega$ of output is paid to labor, with the remaining going to capital. The wage rate in country $i$ is then

$$W_{i,t} = \omega A_{i,t} (K_{i,t})^{1-\omega} \quad i = H, F \tag{3}$$

Capital is supplied by a competitive installment firm. In period $t$ the installment firm produces $I_{i,t}$ units of new capital and sells them at a price $Q_{i,t}$ that the firm takes as given. The production of $I_{i,t}$ units of capital good requires purchasing $I_{i,t}$ units of the consumption good and incurring a quadratic adjustment cost, so the total cost in units of the consumption good is:

$$I_{i,t} + \frac{\xi}{2} \frac{(I_{i,t} - \delta K_{i,t})^2}{K_{i,t}} \tag{4}$$

The profit of installing $I_{i,t}$ units of capital in country $i$ is then $Q_{i,t} I_{i,t}$ minus the cost (4). Profit maximization by the installment firm implies a standard Tobin's Q relation:

$$\frac{I_{i,t}}{K_{i,t}} = \delta + \frac{Q_{i,t} - 1}{\xi} \tag{5}$$

A unit of Home equity is a claim on a unit of Home capital. The equity price is equal to the cost of purchasing one unit of capital from the installment firm, $Q_{H,t}$. An investor purchasing a unit of Home equity at the end of period $t$ gets a dividend of $(1 - \omega) Y_{H,t+1} / K_{H,t+1}$ in period $t+1$, and can sell the remaining $1 - \delta$ units of equity at a price $Q_{H,t+1}$. The returns on Home and Foreign equity are then

$$R_{H,t+1} = \frac{(1 - \omega) A_{H,t+1} (K_{H,t+1})^{-\omega} + (1 - \delta) Q_{H,t+1}}{Q_{H,t}} \tag{6}$$

$$R_{F,t+1} = \frac{(1 - \omega) A_{F,t+1} (K_{F,t+1})^{-\omega} + (1 - \delta) Q_{F,t+1}}{Q_{F,t}} \tag{7}$$

## 2.2 Private Information and Noise

NRE models contain two key elements: private information about future fundamentals and noise that prevents asset prices from completely revealing the private

information. We introduce these elements to the model as follows.

**Private Information**

Each agent receives private signals about next period's productivity innovations in both countries. The signals observed by Home investor $j$ about respectively the log of Home and Foreign productivity are:

$$v_{j,t}^{H,H} \;=\; \varepsilon_{H,t+1} + \epsilon_{j,t}^{H,H} \qquad \epsilon_{j,t}^{H,H} \sim N\left(0, \sigma_{HH}^2\right) \qquad (8)$$
$$v_{j,t}^{H,F} \;=\; \varepsilon_{F,t+1} + \epsilon_{j,t}^{H,F} \qquad \epsilon_{j,t}^{H,F} \sim N\left(0, \sigma_{HF}^2\right) \qquad (9)$$

Each signal consists of the true innovation and a stochastic error. Similarly, agent $j$ in the Foreign country observes the signals:

$$v_{j,t}^{F,H} \;=\; \varepsilon_{H,t+1} + \epsilon_{j,t}^{F,H} \qquad \epsilon_{j,t}^{F,H} \sim N\left(0, \sigma_{HF}^2\right) \qquad (10)$$
$$v_{j,t}^{F,F} \;=\; \varepsilon_{F,t+1} + \epsilon_{j,t}^{F,F} \qquad \epsilon_{j,t}^{F,F} \sim N\left(0, \sigma_{HH}^2\right) \qquad (11)$$

As is standard in NRE models, we assume that the errors of the signals average to zero across investors in a given country ($\int_0^1 \epsilon_{j,t}^{H,H} dj = \int_0^1 \epsilon_{j,t}^{H,F} dj = 0$).

For simplicity we assume that the variance of signals on domestic productivity is the same for agents in the two countries, as is the variance of signals on productivity abroad. We allow for an information asymmetry with agents receiving more precise signals about shocks in their own country than abroad: $\sigma_{HH}^2 \leq \sigma_{HF}^2$.

**Noise**

Noise takes the form of unobserved portfolio shifts between assets for reasons unrelated to expected returns. In the NRE literature the noise is usually simply introduced exogenously in the form of noise trade or liquidity trade. Some papers have introduced it endogenously in various forms of hedge trade and liquidity trade.[4] For our purposes the existence of a source of noise is more important than the exact nature of it.

We introduce the noise through a time-varying cost of investing abroad. This cost plays the dual role of generating portfolio home bias in the steady state of the model. A Home agent $j$ investing in the Foreign country receives the return (7) times an iceberg cost $e^{-\tau_{Hj,t}} < 1$. Similarly, a Foreign agent $j$ investing in the

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[4]See for example Bacchetta and van Wincoop (2006), Dow and Gorton (1995), Spiegel and Subrahmanyam (1992) and Wang (1994).

Home country receives the return (6) times an iceberg cost $e^{-\tau_{Fj,t}} < 1$. The cost of investment abroad does not represent a loss in resources but is instead a fee paid to brokers from the investor's country.

We assume that the average cost across all agents from both countries is constant at $\tau$. The average cost in the Home country is $\tau_{H,t} = \tau(1 + \epsilon_t^\tau)$, where $\epsilon_t^\tau$ has a $N(0, \theta\sigma_a^2)$ distribution. The average cost in the Foreign country is $\tau_{F,t} = \tau(1 - \epsilon_t^\tau)$. Agents can observe their own cost but not the average cost in the country.[5] An increase in $\tau_t^D = \tau_{H,t} - \tau_{F,t} = 2\tau\epsilon_t^\tau$ leads to a portfolio shift towards Home equity. Such unobserved portfolio shifts prevent the relative equity price from revealing private information.

## 2.3   Consumption and Portfolio Choice

Our assumption of an overlapping generation structure simplifies the model in two ways. First, it removes the well-known pitfall in open economy models that temporary income shocks can have a permanent effect on the distribution of wealth across countries when agents have infinite lives. Second, investors have only a one period investment horizon and therefore do not face the issue of hedging against changes in future expected returns.

A young Home agent $j$ at time $t$ chooses her consumption and portfolio to maximize

$$\frac{\left(C_{y,t}^{Hj}\right)^{1-\gamma}}{1-\gamma} + \beta E_t^{Hj} \frac{\left(C_{o,t+1}^{Hj}\right)^{1-\gamma}}{1-\gamma} \tag{12}$$

where $C_{y,t}$ is consumption when young and $C_{o,t+1}$ is consumption when old. We assume $\gamma > 1$. Agent $j$ maximizes (12) subject to the budget constraint and portfolio return

$$C_{o,t+1}^{Hj} = (W_{H,t} - C_{y,t}^{Hj})R_{t+1}^{p,Hj}$$
$$R_{t+1}^{p,Hj} = z_{Hj,t}R_{H,t+1} + (1 - z_{Hj,t})e^{-\tau_{Hj,t}}R_{F,t+1} \tag{13}$$

where $z_{Hj,t}$ is the fraction invested in Home equity.

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[5]More precisely, we assume that the individual cost is an infinitely noisy signal of the average cost. This assumption can be relaxed but simplifies the analysis.

The first-order conditions for consumption and portfolio choice are:

$$\left(C_{y,t}^{Hj}\right)^{-\gamma} = \beta \left(W_{H,t} - C_{y,t}^{Hj}\right)^{-\gamma} E_t^{Hj} \left(R_{t+1}^{p,Hj}\right)^{1-\gamma} \tag{14}$$

$$E_t^{Hj} \left(R_{t+1}^{p,Hj}\right)^{-\gamma} \left(R_{H,t+1} - R_{F,t+1}e^{-\tau_{Hj,t}}\right) = 0 \tag{15}$$

Optimal portfolio allocation equates the expected discounted return (the expected product of the asset pricing kernel and asset returns) across assets. The asset pricing kernel is the marginal utility of future consumption, which is proportional to the return on the agent's portfolio.

Foreign agents face an analogous decision problem with portfolio return

$$R_{t+1}^{p,Fj} = z_{Fj,t}e^{-\tau_{Fj,t}}R_{H,t+1} + (1 - z_{Fj,t})R_{F,t+1} \tag{16}$$

The corresponding optimality conditions for a Foreign investor $j$ are:

$$\left(C_{y,t}^{Fj}\right)^{-\gamma} = \beta \left(W_{F,t} - C_{y,t}^{Fj}\right)^{-\gamma} E_t^{Fj} \left(R_{t+1}^{p,Fj}\right)^{1-\gamma} \tag{17}$$

$$E_t^{Fj} \left(R_{t+1}^{p,Fj}\right)^{-\gamma} \left(R_{H,t+1}e^{-\tau_{Fj,t}} - R_{F,t+1}\right) = 0 \tag{18}$$

The average portfolio shares invested by Home and Foreign investors in Home equity are denoted $z_{H,t} = \int_0^1 z_{Hj,t}dj$ and $z_{F,t} = \int_0^1 z_{Hj,t}dj$.

## 2.4   Asset and Goods Market Clearing

We assume that the brokers who receive the fees on investment abroad fully consume it. Owners of the installment firms also consume profits each period. The goods market equilibrium condition is

$$Y_{H,t+1} + Y_{F,t+1} = Q_{H,t+1}I_{H,t+1} + Q_{F,t+1}I_{F,t+1} + \int_0^1 C_{y,t+1}^{Hj}dj + \int_0^1 C_{y,t+1}^{Fj}dj$$

$$+ \int_0^1 \left(W_{H,t} - C_{y,t}^{Hj}\right)\left(z_{Hj,t}R_{H,t+1} + (1 - z_{Hj,t})R_{F,t+1}\right)dj$$

$$+ \int_0^1 \left(W_{F,t} - C_{y,t}^{Fj}\right)\left(z_{Fj,t}R_{H,t+1} + (1 - z_{Fj,t})R_{F,t+1}\right)dj$$

The left hand side is world output. The first two terms on the right hand side represent investment.[6] The next two terms represent consumption by young agents.

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[6]The installation cost does not enter. On the one hand it raises demand for the good (from the installation process itself). On the other hand it reduces profits, and therefore consumption, of the owners of installment firms.

The final two terms represent consumption by old agents and the brokers.[7]

Asset market clearing requires that the value of capital in a country is equal to the value of holdings of the country's equity by young agents. The financial wealth of respectively a Home and Foreign agent $j$ is $W_{Ht} - C_{y,t}^{Hj}$ and $W_{Ft} - C_{y,t}^{Fj}$. The asset market clearing conditions are then

$$Q_{H,t}K_{H,t+1} = \int_0^1 (W_{Ht} - C_{y,t}^{Hj})z_{Hj,t}dj + \int_0^1 (W_{Ft} - C_{y,t}^{Fj})z_{Fj,t}dj$$

$$Q_{F,t}K_{F,t+1} = \int_0^1 (W_{Ht} - C_{y,t}^{Hj})(1 - z_{Hj,t})dj + \int_0^1 (W_{Ft} - C_{y,t}^{Fj})(1 - z_{Fj,t})dj$$

# 3  Solution Method

Figure 1 puts the model in perspective in context of the literature and illustrates why there is no off the shelve solution. The model has the following key features: (i) information dispersion, (ii) portfolio choice, (iii) general equilibrium nature and (iv) non-linearity. NRE models have the first two features, but not the last two. The absence of non-linearity and general equilibrium aspects significantly simplifies the solution of these models. On the other hand, most DSGE models in macro and open economy macro only have the last two features. Only recently has the literature begun to investigate DSGE open economy models with portfolio choice and to develop methods for solving them.[8] But even these models do not contain information dispersion.

The solution we develop combines and extends methods for solving standard NRE models with a recently developed local approximation methods for solving DSGE models with portfolio choice. NRE models are usually solved in three steps. The first step involves a conjecture for the equilibrium asset price. The second step computes the expectation of future asset payoffs by solving a signal extraction problem that uses public and private information as well as information from the equilibrium asset price. The last step invokes asset market equilibrium. The main difficulty here will be in the last step as we need to impose not just asset market equilibrium but the complete general equilibrium of the model in a

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[7]The cost of investing abroad does not enter, as the income of the brokers exactly offsets the cost for old agents.

[8]See Devereux and Sutherland (2007), Evans and Hnatkovska (2007) and Tille and van Wincoop (2008).

highly non-linear environment. We will do so by extending the local approximation method recently developed by Devereux and Sutherland (2007) and Tille and van Wincoop (2008) for DSGE models with portfolio choice.

We will discuss each of these three steps in broad terms, leaving algebraic details to the Appendix and the Technical Appendix that is available on request. We use lower case letters for logs and superscripts A and D to denote respectively the average and difference of a variable across the two countries ($x^D = x_H - x_F$, $x^A = (x_H + x_F)/2$).

## 3.1 Asset Price Conjecture

Only the relative asset price will be affected by private information. The average asset price is driven by global asset demand and therefore global saving. The latter is not affected by private information, but we will consider an extension where it will be. We make the following conjecture for the relative log asset price $q_t^D = q_{H,t} - q_{F,t}$:

$$q_t^D = f(S_t, x_t^D) \tag{19}$$

where

$$S_t = (a_t^D, a_t^A, k_t^D, k_t^A) \tag{20}$$

is the vector of observed state variables and

$$x_t^D = \varepsilon_{t+1}^D + \lambda \tau_t^D / \tau \tag{21}$$

is an unobserved state variable. Since we will adopt a local approximation method, described below, the conjecture (19) will be verified locally up to quadratic terms in observed and unobserved state variables.

The logic behind this conjecture is as follows. As in any DSGE model, the solution for control variables (including asset prices) will be a function of state variables. Usually these state variables are observed. In our model this is the case for the variables $S_t$. However, there are now also unobserved state variables, which are conjectured to jointly affect the asset price through $x_t^D$. The relative future productivity innovation $\varepsilon_{t+1}^D$ should affect the relative asset price through private information. The relative asset price should depend on $\tau_t^D$ as time variation in this relative friction leads to portfolio shifts between Home and Foreign equity.

## 3.2 Signal Extraction

This conjecture significantly simplifies signal extraction. While the function $f(.)$ will be non-linear in $x_t^D$, two aspects make simple linear signal extraction feasible. First, we have conjectured (and will verify) that the relative asset price depends on a variable $x_t^D$ that is linear in the unknowns $\varepsilon_{t+1}^D$ and $\tau_t^D$. Second, locally $q_t^D$ will depend on $x_t^D$ with a positive slope. This means that we can extract $x_t^D$ from knowledge of the relative asset price $q_t^D$, and the observed state space $S_t$. The asset price signal therefore translates into a signal that is linear in the future fundamental $\epsilon_{t+1}^D$ and the "noise" $\tau_t^D$.

We then have three linear signals about next period's technology innovations: (i) the price signal, which tells us the level of $\varepsilon_{t+1}^D + \lambda \tau_t^D / \tau$, (ii) the private signals and (iii) the public signals that $\varepsilon_{H,t+1}$ and $\varepsilon_{F,t+1}$ are drawn from independent $N(0, \sigma_a^2)$ distributions. We solve this signal extraction problem in Appendix B. It gives conditional normal distributions of $\varepsilon_{H,t+1}$ and $\varepsilon_{F,t+1}$ that vary across agents. The expectation of future productivity innovations by agent $j$ in the Home country takes the form

$$E_t^{H,j} \begin{pmatrix} \varepsilon_{H,t+1} \\ \varepsilon_{F,t+1} \end{pmatrix} = \begin{pmatrix} \alpha_{x,H} x_t^D + \alpha_{HH} v_{j,t}^{H,H} + \alpha_{HF} v_{j,t}^{H,F} \\ -\alpha_{x,F} x_t^D + \alpha_{FH} v_{j,t}^{H,H} + \alpha_{FF} v_{j,t}^{H,F} \end{pmatrix} \tag{22}$$

All coefficients are positive and are defined in the Appendix. The average expectation across Home agents is then

$$\bar{E}_t^H \begin{pmatrix} \varepsilon_{H,t+1} \\ \varepsilon_{F,t+1} \end{pmatrix} = \begin{pmatrix} (\alpha_{x,H} + \alpha_{HH})\varepsilon_{H,t+1} + (\alpha_{HF} - \alpha_{x,H})\varepsilon_{F,t+1} + \alpha_{x,H} \lambda \tau_t^D / \tau \\ (\alpha_{FH} - \alpha_{x,F})\varepsilon_{H,t+1} + (\alpha_{x,F} + \alpha_{FF})\varepsilon_{F,t+1} - \alpha_{x,F} \lambda \tau_t^D / \tau \end{pmatrix} \tag{23}$$

Analogous results apply to Foreign agents. Average expectations about future productivity therefore depend on future productivity levels themselves and on the noise $\tau_t^D$. Through rational confusion an increases in $\tau_t^D$ raises the expectation of $\epsilon_{t+1}^D$. This is because a rise in $\tau_t^D$ leads to a higher relative price of Home equity, which agents use as a signal of future relative productivity.

## 3.3 General Equilibrium

The final step in the solution of NRE models involves imposing asset market equilibrium. In a DSGE model this step is more involved since we will need to invoke the full general equilibrium of the model, including multiple asset market and

goods market clearing conditions and Euler equations for portfolio choice and consumption. Moreover, we need to do so in a highly non-linear environment.

We adopt and extend the local approximation method for DSGE models with portfolio choice developed by Devereux and Sutherland (2007) and Tille and van Wincoop (2008), from hereon DS and TvW. It provides an exact solution to the zero, first and second-order components of control and state variables. The only exception is $z_t^D = z_{H,t} - z_{F,t}$, for which the method delivers the zero and first-order components. A variable can always be decomposed into its components of all orders. The zero-order component of $x_t$ is the level of $x$ when $\sigma_a \to 0$, denoted $x(0)$. The first-order component $x_t(1)$ is linear in model innovations or in the standard deviation $\sigma_a$ of model innovations. Higher orders are defined analogously.

The method distinguishes between the difference across countries in portfolio Euler equations and all "other equations" and similarly between the difference $z_t^D$ across countries in portfolio allocation and all "other variables". It first solves for the zero-order component of $z^D$ and the first-order component of the "other variables" by jointly imposing the second-order component of the difference across countries in portfolio Euler equations and the first-order component of the "other equations". This step is subsequently repeated one order higher for all equations and variables in order to obtain the first-order component of $z_t^D$ jointly with the second-order component of all "other variables". We refer to DS and TvW for detailed descriptions of the method.

In implementing and extending the method to our model, three issues need to be addressed that are specific to the introduction of information dispersion. These involve the order component of the errors of the private signals, the computation of expectations of equations and the computation of the parameter $\lambda$ that captures the noise to signal ratio in the relative asset price in equation (21).

**Errors in Private Signals**

We assume that the parameters $\sigma_{HH}^2$ and $\sigma_{HF}^2$ (variance of errors of private signals) are zero-order. In order to avoid an explosion of portfolio shares when risk becomes small we need differences in expected returns to be relatively small, of order two or higher.[9] This will indeed be the case under the assumption above. As the errors of the private signals are large, of order zero, the weight given to the

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[9]This is because expected returns are divided by the variance of the excess return in the optimal portfolios.

private signals in expectations of future productivity innovations is small, of order two and higher. This leads to differences in expectations of order two and higher. If errors in private signals were first-order, differences in expectations would be first-order as well and relative portfolios would explode for low levels of risk. For the same reason we will also assume that the average cost $\tau$ of investment abroad is second-order.

**Computing Expectations**

Consider the expected value of a term $eq$, which consists of one or several variables, $E \ eq$. In common knowledge models, computing the second-order term of this expectation simply entails taking the expectation of the second-order component of $eq$, so that $[E \ eq](2) = E[eq(2)]$. This is no longer the case here though.[10] We need to be careful to first compute expectations of equations before computing order components. In order to compute expectations of equations, both the equations and the solution of control variables need to be in polynomial form. It is sufficient to use an $o$-order polynomial approximation when the goal is to compute the $o$-order component of an equation or variable.

Equations are written as polynomials in $S_t$, $x_t^D$, $x_{t+1}^D$ and $\epsilon_{t+1} = (\epsilon_{H,t+1}, \epsilon_{F,t+1})$. Control variables are conjectured as polynomial solutions in the observed and unobserved state variables $S_t$ and $x_t^D$. A quadratic polynomial conjecture for the control variables is sufficient as we will only solve zero, first and second-order components of control variables. We therefore conjecture[11]

$$q_t^h = \alpha_h S_t + S_t' A_h S_t + \eta_h x_t^D + \phi_h S_t x_t^D + \mu_h \left(x_t^D\right)^2 \quad h = D, A \qquad (24)$$

$$c_{yt}^h = \alpha_{y,h} S_t + S_t' A_{y,h} S_t + \eta_{y,h} x_t^D + \phi_{y,h} S_t x_t^D + \mu_{y,h} \left(x_t^D\right)^2 \quad h = D, A \quad (25)$$

$$k_{t+1}^h = \alpha_{k,h} S_t + S_t' A_{k,h} S_t + \eta_{k,h} x_t^D + \phi_{k,h} S_t x_t^D + \mu_{k,h} \left(x_t^D\right)^2 \quad h = D, A \quad (26)$$

$$z_t^h = \alpha_{z,h} S_t + S_t' A_{z,h} S_t + \eta_{z,h} x_t^D + \phi_{z,h} S_t x_t^D + \mu_{z,h} \left(x_t^D\right)^2 \quad h = A \qquad (27)$$

Expectations of equations are computed using the results from signal extraction. Invoking the order components of equations as in DS and TvW will then give the zero and first-order components of the parameters $\alpha$ and $\eta$ (with various subscripts) in (24)-(27) and the zero-order component of all the other parameters.

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[10] As an example, $\epsilon_{H,t+1}(2) = 0$, so that $E_t[\epsilon_{H,t+1}(2)] = 0$. But $E_t(\epsilon_{H,t+1})$ has a non-zero second-order component as the weight attached to private signals is of order two and higher.

[11] No conjecture will be needed for $z_t^D$. After all "other variables" are solved up to second order, $z_t^D(1)$ follows from the third-order component of the difference in portfolio Euler equations.

**Computing $\lambda$**

In NRE models the signal to noise ratio $\lambda$ can be solved by imposing asset market equilibrium. A version of that applies here as well. We need to impose the difference between the two asset market clearing conditions. This relates the average share invested in Home equity, $z_t^A$, to the share of Home equity supply. The solution discussed so far solves $z_t^A(1)$ by equating it to the first-order component from the supply side. In order to actually impose market equilibrium we need to compute $z_t^A(1)$ from a portfolio or demand perspective as well. This is done by using the third-order component of the average of the Euler equations for portfolio choice. Leaving the algebraic details to the Appendix, equating $z_t^A(1)$ from the demand side to the Home equity share from the supply side yields a solution for $\lambda$.

# 4 Asset Prices, Portfolio Allocation and Capital Flows

In this section we discuss the first-order solution of asset prices, optimal portfolio shares and capital flows.

## 4.1 Asset Prices

The first-order solution of the relative asset price is

$$
\begin{aligned}
q_t^D(1) &= \alpha_D(0)S_t(1) + \eta_D(0)x_t^D(1) \\
&= \alpha_{D,1}(0)a_t^D + \alpha_{D,3}(0)k_t^D(1) + \eta_D(0)\varepsilon_{t+1}^D + \eta_D(0)\lambda\tau_t^D(3)/\tau \quad (28)
\end{aligned}
$$

with all parameters positive. The relative asset price is therefore driven by both observable fundamentals, $a_t^D$ and $k_t^D$, and by unobservables $\varepsilon_{t+1}^D$ and $\tau_t^D$. Both of these unobservables generate a disconnect between asset prices and observed fundamentals that is widely documented.

In the absence of information dispersion the relative asset price would, to the first-order, be entirely determined by the observed fundamentals $S_t$. This can be seen as follows. First, in the absence of private information future productivity innovations cannot affect equilibrium asset prices. Second, shocks to $\tau_t^D$ only have a third-order effect on asset prices. A rise in $\tau_t^D = 2\tau\epsilon_t^\tau$ is third-order. This leads

to a third-order increase in the expected excess return on Home equity, which generates a first-order shift of portfolios towards Home equity. In order to clear financial markets there will be a third-order rise in the Home equity price, leading to a third-order drop in the expected excess return on Home equity.

At first it may seem surprising that both $\tau_t^D$ and $\epsilon_{t+1}^D$ have a first-order effect on asset prices when we introduce information dispersion. After all, shocks to $\tau_t^D$ are third-order and private information alone leads to changes in average expectations about $\epsilon_{t+1}^D$ that are third-order as well: a second-order weight attached to private signals times $\epsilon_{t+1}^D$. However, this ignores the role of the relative asset price as an information coordination mechanism. Imagine for a moment that agents ignored $q_t^D$ as a source of information. Then the impact of the unobservables would be third-order as conjectured above, so of the form $\sigma_a^2 \epsilon_{t+1}^D + \kappa \tau_t^D$ with $\kappa$ a zero-order constant. But then they would have a very good signal about $\epsilon_{t+1}^D$ through the relative asset price. Dividing by $\sigma_a^2$ the signal would be $\epsilon_{t+1}^D + \tilde{\kappa}\tau_t^D/\tau$ where $\tilde{\kappa} = \kappa\tau/\sigma_a^2$ is a new zero-order constant. The error in the signal is then first-order. Based on this a change in either $\epsilon_{t+1}^D$ or $\tau_t^D/\tau$ would lead to a first-order change in the expectation of $\epsilon_{t+1}^D$ and therefore a first-order change in the relative asset price $q_t^D$.

## 4.2   Portfolio Allocation

It is useful to discuss the implications of the model for portfolio allocation as this will affect international capital flows. We present the results in terms of the average portfolio share invested in Home equity, $z_t^A$ and the difference across countries in the portfolio share invested in Home equity, $z_t^D$. We consider both their zero and first-order components. Asset market clearing implies that $z^A(0) = 0.5$. The difference in zero-order portfolio shares, which represents portfolio home bias, is computed from the second-order component of the difference in portfolio Euler equations. It is driven by the mean level $\tau$ of international financial frictions:

$$z^D(0) = \frac{2\tau}{\gamma[var_t(er_{t+1})](2)} \tag{29}$$

We obtain expressions for the first-order component of the average and difference in optimal portfolio shares from the third-order component of respectively

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the average and difference in portfolio Euler equations:

$$z_t^A(1) = \frac{\tau_t^D(3)}{\gamma[var_t(er_{t+1})](2)} + \frac{[\bar{E}_t er_{t+1}(3)](3)}{\gamma[var_t(er_{t+1})](2)} - \tag{30}$$

$$(\gamma - 1)\frac{[var_t(r_{H,t+1}) - var_t(r_{F,t+1})](3)}{\gamma[var_t(er_{t+1})](2)} + \frac{(1-\gamma)^2}{2}\frac{[\bar{E}_t(r_{t+1}^A)^2 er_{t+1}](3)}{\gamma[var_t(er_{t+1})](2)}$$

$$z_t^D(1) = \frac{[\bar{E}_{H,t}er_{t+1}](3) - [\bar{E}_{F,t}er_{t+1}](3)}{\gamma[var_t(er_{t+1})](2)} - \frac{1}{2}z^D(0)\frac{[var_t(er_{t+1})](3)}{[var_t(er_{t+1})](2)} \tag{31}$$

Here $\bar{E}_t^i$ denotes the average expectation across agents in country $i$ $(i = H, F)$ and $\bar{E}_t$ the average expectation across agents from both countries.

The first-order component of $z_t^A$ is driven by four intuitive elements. First, a rise in $\tau_t^D(3)$ leads to a portfolio shift towards Home equity as the cost of investment abroad rises for Home relative to Foreign investors. Second, a higher average expected excess return $er_{t+1}$ on Home equity net of financial frictions also leads to a portfolio shift towards Home equity.

The last two terms represent time-variation in second moments, which are captured by their third-order components.[12] A rise in the variance of the Home return relative to that of the Foreign equity return leads to a shift towards Foreign equity. When the excess return on Home equity is expected to be high during periods of high global volatility $((r_{t+1}^A)^2$ high$)$, Home equity is a good hedge against such global risks and there is a shift towards Home equity. Using the first and second-order solution for the "other variables", the third-order components of these moments can be computed as a function of both observed and unobserved state variables:

$$[var_t(r_{H,t+1}) - var_t(r_{F,t+1})](3) = \psi_1(x_t^D)^3 + \sigma_a^2\psi_2 x_t^D + \sigma_a^2\psi_3 S_t(1) \tag{32}$$

$$[\bar{E}_t(r_{t+1}^A)^2 er_{t+1}](3) = \psi_4(x_t^D)^3 + \sigma_a^2\psi_5 x_t^D \tag{33}$$

where the parameters $\psi_i$ are zero-order coefficients.

The expression (31) for the difference $z_t^D(1)$ in portfolio shares captures time-variation in portfolio home bias. It is driven by two factors. First, an increase in the expected excess return on Home equity by Home investors relative to Foreign investors will lead to increased home bias. Second, an increase in the variance of the excess return leads to an increased incentive for diversification, which reduces home bias.

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[12]See Tille and van Wincoop (2008) for a further discussion of this.

We obtain the following solution for these moments

$$[\bar{E}_{H,t}er_{t+1}](3) - [\bar{E}_{F,t}er_{t+1}](3) = \delta_4\sigma_a^2 \left[\frac{1}{\sigma_{HH}^2} - \frac{1}{\sigma_{HF}^2}\right] \varepsilon_{t+1}^A \tag{34}$$

$$[var_t(er_{t+1})](3) = \delta_1(x_t^D)^3 + \sigma_a^2\delta_2 x_t^D + \sigma_a^2\delta_3 S_t(1) \tag{35}$$

where the parameters $\delta_i$ are zero-order and follow from the first and second-order solutions of the "other variables". To understand (34), assume that $\sigma_{HH}^2 < \sigma_{HF}^2$, so that agents have better quality signals about their domestic equity market. When productivity levels rise in both countries next period, agents from both countries expect that productivity in their own country will rise more because they have better quality information about their own productivity. As a result they both expect the return on their own country's equity to rise relative to that of the other country, which leads to increased portfolio home bias ($\delta_4 > 0$). (35) implies that changes in the variance of the excess return over time are driven by both changes in observed and unobserved state variables.

## 4.3   International Capital Flows

After some straightforward balance of payments accounting, and using the results on portfolio allocation discussed above, we obtain the following expressions for capital outflows and inflows:

$$
\begin{aligned}
outflows_t(1) &= (1 - z_H(0))\, s_t^H(1) + \frac{z^D(0)}{4}\frac{\Delta[var_t(er_{t+1})](3)}{[var_t(er_{t+1})](2)} \\
&\quad - \frac{\left(\Delta\left[\bar{E}_t er_{t+1}\right](3)\right)^{IS}}{\gamma[var_t(er_{t+1})](2)} - \frac{1}{2}\frac{\Delta[\bar{E}_{H,t}er_{t+1}](3) - \Delta[\bar{E}_{F,t}er_{t+1}](3)}{\gamma(var_t(er_{t+1})(2)} \\
inflows_t(1) &= (1 - z_H(0))\, s_t^F(1) + \frac{z^D(0)}{4}\frac{\Delta[var_t(er_{t+1})](3)}{[var_t(er_{t+1})](2)} \\
&\quad + \frac{\left(\Delta\left[\bar{E}_t er_{t+1}\right](3)\right)^{IS}}{\gamma[var_t(er_{t+1})](2)} - \frac{1}{2}\frac{\Delta[\bar{E}_{H,t}er_{t+1}](3) - \Delta[\bar{E}_{F,t}er_{t+1}](3)}{\gamma(var_t(er_{t+1})(2)}
\end{aligned}
$$

(36)

(37)

The terms on the right hand side are related to saving, expected returns and risk. For each of them we now discuss their intuitive meaning and determinants.

**Portfolio Growth**

The first term represents portfolio growth, which measures outflows and inflows when Home and Foreign saving are invested abroad at the steady state portfolio

share $1 - z_H(0)$. The portfolio growth component depends entirely on Home and Foreign saving, which can be written as

$$s_t^H(1) = \alpha_H \Delta S_t(1) - 0.5z^D(0)\Delta q_t^D(1) \tag{38}$$

$$s_t^F(1) = \alpha_F \Delta S_t(1) + 0.5z^D(0)\Delta q_t^D(1) \tag{39}$$

where $\alpha_H$ and $\alpha_F$ are zero-order vectors. Home and Foreign saving depend both on changes in observed state variables and changes in relative asset prices. The latter generate wealth effects that impact consumption of the old generations. When the relative price of Home equity rises, the old generation in the Home country will be relatively wealthy and will consume this additional wealth. This lowers Home saving.

**Time-Varying Risk**

The other three terms driving capital inflows and outflows are a result of portfolio reallocation due to changes in risk and expected returns. The second term represents capital flows due to changes in the variance of the excess return. An increase in the variance of the excess return makes portfolio diversification more attractive and therefore leads to an increase in both capital inflows and outflows. As can be seen from (35), the variance of the excess return depends on both observed and unobserved state variables. Time variation in the other second moments, shown in (32) and (33), does not affect capital flows. These moments affect average portfolio shares. When there is an average shift towards Home equity, the market will equilibrate through a third-order rise in the relative Home equity price. This leads to a third-order drop in the expected excess return on Home equity, causing a first-order portfolio shift back towards Foreign equity.[13] In the end capital flows remain unaffected.

**Average Expected Excess Return**

The third term on the right hand side of (36) and (37) represents capital flows due to the average change in the expected excess return. As discussed in detail in Tille and van Wincoop (2008), not all changes in expected excess returns generate capital flows. We have already discussed the example above where changes in expected returns equilibrate asset markets when there are time-varying second

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[13]As can be seen from (30) and (31), third-order changes in expected returns lead to first-order portfolio shifts as they are divided by a second-order variance of the excess return.

moments. No capital flows result from this. Another example is a rise in the expected excess return on Home equity needed to equilibrate asset markets when a higher relative Home equity price raises the relative supply of Home equity. It can be shown that no capital flows result from this either as even without asset trade the portfolio share invested in Home equity rises due to valuation effects.

The technical appendix derives all of the components determining changes in the equilibrium expected excess return. The only one that affects capital flows is denoted with an $IS$ superscript in (36) and (37). It is related to changes in saving and investment and is equal to

$$\Delta \bar{E}_t er_{t+1}(3)^{IS} = \frac{\gamma[var_t(er_{t+1})](2)}{4} \left[ i_t^D(1) - z^D(0)s_t^D(1) \right] \tag{40}$$

When relative investment is high in the Home country it raises the relative supply of Home equity. A higher expected excess return on Home equity is then needed to clear asset markets. This leads to increased capital inflows and lower capital outflows. When relative saving in Home is high, there will be an excess demand for Home equity due to portfolio home bias. A lower expected excess return is then needed to clear asset markets, which leads to larger outflows and smaller inflows.

Cross-country differences in saving and investment are equal to

$$s_t^D(1) = \Delta a_t^D(1) + (1-\omega)\Delta k_t^D(1) - z^D(0)\Delta q_t^D(1) \tag{41}$$

$$i_t^D(1) = \frac{1}{\xi}q_t^D(1) \tag{42}$$

Relative asset prices affect relative saving through a wealth effect and relative investment through a standard Tobin's Q equation.

**Differences in Expected Returns across Countries**

The last term driving capital outflows and inflows in (36) and (37) represents changes in the average expected excess return of Home investors relative to Foreign investors. When investors from both countries become more optimistic about the expected excess return on their domestic equity, both capital outflows and inflows will drop. As can be seen from (34), this will happen when there is a positive future world productivity innovation $\epsilon_{t+1}^A$ and investors have better quality information about domestic productivity innovations. Investors from both countries then believe that their own relative productivity will rise as they have better information on that, leading to a retrenchment towards domestic assets.

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**Summary**

Figure 2 summarizes the role that information dispersion plays in determining capital flows. In the absence of information dispersion capital flows are entirely determined by the observed state variables $S_t$. Figure 2 illustrates the four channels through which unobserved state variables affect capital flows in the presence of information dispersion. The first two channels take place through the relative asset price $q_t^D$. We have already seen that the unobservables $\tau_t^D$ and $\epsilon_{t+1}^D$ have a first-order effect on asset prices, which affect saving and investment through a wealth effect and a Tobin's Q effect. This in turn affects capital flows both through changes in the equilibrium expected excess return and through portfolio growth. The third channel is through the impact of the unobserved state variables $\tau_t^D$ and $\epsilon_{t+1}^D$ on the variance of the excess return. Finally, the third unobserved state variable, $\epsilon_{t+1}^A$, affects capital flows through changes in the expected excess return of Home relative to Foreign investors.

While gross capital flows are affected by the unobserved state variables through each of these four channels, net capital flows are only affected by the first two. Taking the difference between (36) and (37), net capital flows are driven by portfolio growth and changes in the expected excess return due to saving and investment. One can rewrite the sum of these terms as simply saving minus investment. This follows by identity as net capital outflows are equal to the current account. Therefore

$$outflows_t(1) - inflows_t(1) = s_t^H(1) - i_t^H(1) = \frac{1}{2}\left[s_t^D(1) - i_t^D(1)\right]$$

Information dispersion therefore affects net capital flows to the extent that it affects relative asset prices, which in turn affect relative saving and investment.

These results for capital flows lead to two qualitative implications that will be brought to the data in the next section. Each of these implications is the result of information dispersion. They apply to both gross and net capital flows. The implications, which follow immediately from Figure 2, are

**Implication 1** *Capital flows are partially disconnected from observed macro fundamentals.*

**Implication 2** *After conditioning on observed macro fundamentals, capital flows contain information about future macro fundamentals.*

# 5  Empirical Results

To be written

# 6  Conclusion

To be written

# Appendix

# A    Equations of the model

The various equations of the model can be written in terms of the logs of the various variables, denoted by lower-case letters. We denote the worldwide average of log equity prices by $q_t^A = 0.5\,(q_{H,t} + q_{F,t})$, and the cross-country difference in log equity prices by $q_t^D = q_{H,t} - q_{F,t}$. We define similar variables for the capital stock $(k_{t+1}^A,\ k_{t+1}^D)$, productivity $(a_t^A,\ a_t^D)$ and asset returns $(r_{t+1}^A,\ r_{t+1}^D = er_{t+1})$.

The Tobin's Q (5) in Home and Foreign are:

$$e^{k_{t+1}^A + \frac{1}{2}k_{t+1}^D} = \left(1 + \frac{e^{q_t^A + \frac{1}{2}q_t^D} - 1}{\xi}\right) e^{k_t^A + \frac{1}{2}k_t^D} \tag{43}$$

$$e^{k_{t+1}^A - \frac{1}{2}k_{t+1}^D} = \left(1 + \frac{e^{q_t^A - \frac{1}{2}q_t^D} - 1}{\xi}\right) e^{k_t^A - \frac{1}{2}k_t^D} \tag{44}$$

The consumption Euler equations (14) and (17) are:

$$\left(\omega e^{a_t^A + \frac{1}{2}a_t^D + (1-\omega)\left(k_t^A + \frac{1}{2}k_t^D\right) - c_{yt}^{Hj}} - 1\right)^{\gamma} = \beta E_t^{Hj} e^{(1-\gamma)r_{t+1}^{p,Hj}} \tag{45}$$

$$\left(\omega e^{a_t^A - \frac{1}{2}a_t^D + (1-\omega)\left(k_t^A - \frac{1}{2}k_t^D\right) - c_{yt}^{Fj}} - 1\right)^{\gamma} = \beta E_t^{Fj} e^{(1-\gamma)r_{t+1}^{p,Fj}} \tag{46}$$

The portfolio Euler equations for individual investors (15) and (18) are:

$$0 = E_t^{Hj}\left(e^{-\gamma r_{t+1}^{p,Hj} + r_{t+1}^A + \frac{1}{2}er_{t+1}} - e^{-\gamma r_{t+1}^{p,Hj} - \tau_{Hj,t} + r_{t+1}^A - \frac{1}{2}er_{t+1}}\right) \tag{47}$$

$$0 = E_t^{Fj}\left(e^{-\gamma r_{t+1}^{p,Fj} - \tau_{Fj,t} + r_{t+1}^A + \frac{1}{2}er_{t+1}} - e^{-\gamma r_{t+1}^{p,Fj} + r_{t+1}^A - \frac{1}{2}er_{t+1}}\right) \tag{48}$$

The asset market clearing conditions are:

$$e^{k_{t+1}^A + \frac{1}{2}k_{t+1}^D + q_t^A + \frac{1}{2}q_t^D} = \int\left(\omega e^{a_t^A + \frac{1}{2}a_t^D + (1-\omega)\left(k_t^A + \frac{1}{2}k_t^D\right)} - e^{c_{yt}^{Hj}}\right) z_{Hj,t}\,dj \tag{49}$$

$$+ \int\left(\omega e^{a_t^A - \frac{1}{2}a_t^D + (1-\omega)\left(k_t^A - \frac{1}{2}k_t^D\right)} - e^{c_{yt}^{Fj}}\right) z_{Fj,t}\,dj$$

$$e^{k_{t+1}^A - \frac{1}{2}k_{t+1}^D + q_t^A - \frac{1}{2}q_t^D} = \int\left(\omega e^{a_t^A + \frac{1}{2}a_t^D + (1-\omega)\left(k_t^A + \frac{1}{2}k_t^D\right)} - e^{c_{yt}^{Hj}}\right) (1 - z_{Hj,t})\,dj \tag{50}$$

$$+ \int\left(\omega e^{a_t^A - \frac{1}{2}a_t^D + (1-\omega)\left(k_t^A - \frac{1}{2}k_t^D\right)} - e^{c_{yt}^{Fj}}\right) (1 - z_{Fj,t})\,dj$$

The rates of returns on Home and Foreign equity (6)-(7) are given by:

$$e^{r_{t+1}^A + \frac{1}{2}er_{t+1}} = (1-\omega)\, e^{a_{t+1}^A + \frac{1}{2}a_{t+1}^D - \omega\left(k_{t+1}^A + \frac{1}{2}k_{t+1}^D\right) - q_t^A - \frac{1}{2}q_t^D} \tag{51}$$
$$+ (1-\delta)\, e^{q_{t+1}^A + \frac{1}{2}q_{t+1}^D - q_t^A - \frac{1}{2}q_t^D}$$

$$e^{r_{t+1}^A - \frac{1}{2}er_{t+1}} = (1-\omega)\, e^{a_{t+1}^A - \frac{1}{2}a_{t+1}^D - \omega\left(k_{t+1}^A - \frac{1}{2}k_{t+1}^D\right) - q_t^A + \frac{1}{2}q_t^D} \tag{52}$$
$$+ (1-\delta)\, e^{q_{t+1}^A - \frac{1}{2}q_{t+1}^D - q_t^A + \frac{1}{2}q_t^D}$$

The portfolio returns of individual investors (13)-(16) are:

$$e^{r_{t+1}^{p,Hj}} = z_{Hj,t}e^{r_{t+1}^A + \frac{1}{2}er_{t+1}} + (1-z_{Hj,t})e^{r_{t+1}^A - \frac{1}{2}er_{t+1} - \tau_{Hj,t}} \tag{53}$$

$$e^{r_{t+1}^{p,Fj}} = z_{Fj,t}e^{r_{t+1}^A + \frac{1}{2}er_{t+1} - \tau_{Fj,t}} + (1-z_{Fj,t})e^{r_{t+1}^A - \frac{1}{2}er_{t+1}} \tag{54}$$

The zero order components of the variables are:

$$a(0) = q(0) = 0$$
$$e^{r(0)} = (1-\omega)\, e^{-\omega k(0)} + (1-\delta)$$
$$e^{c_y(0)} = \omega e^{(1-\omega)k(0)} - e^{k(0)}$$

where $k(0)$ solves:

$$\left(\omega e^{-\omega k(0)} - 1\right)^{-\gamma} = \beta\left[(1-\omega)\, e^{-\omega k(0)} + (1-\delta)\right]^{1-\gamma}$$

The ratio of young consumption to the wage is:

$$\bar{c} = \frac{1}{\omega}e^{c_y(0) - (1-\omega)k(0)} = 1 - \frac{e^{\omega k(0)}}{\omega}$$

The average portfolio share is computed from the asset market clearing (49):

$$z^A(0) = \frac{z_H(0) + z_F(0)}{2} = \frac{1}{2}$$

# B   Signal extraction

**General approach**

We focus on the signal extraction of a Home investors. The inferences of a Foreign investors are computed along similar lines.

A Home investor observes the component $x_t^D$ of the equity price differential that is not attributable to the state variables, as well as her private signals on Home

and Foreign future productivity shocks, $v_{j,t}^{H,H}$ and $v_{j,t}^{H,F}$. From (56), $x_t^D$ only has a first order component. The Home investor infers the Home and Foreign future productivity shocks, $\varepsilon_{t+1}^H$ and $\varepsilon_{t+1}^F$ from these signals.

The signal extraction problem therefore consists of inferring a vector $\xi_{t+1} = [\varepsilon_{H,t+1}, \varepsilon_{F,t+1}]'$ conditional on a vector of signals $Y_t^{jh} = \left[x_t^D, v_{j,t}^{H,H}, v_{j,t}^{H,F}\right]'$ which are linked as follows:

$$Y_t^{jh} = H^{h\prime} \xi_{t+1} + w_t^{jh}$$

where $w_t^{jh} = \left[\lambda \frac{\tau_t^D}{\tau}, \epsilon_{j,t}^{H,H}, \epsilon_{j,t}^{H,F}\right]'$ are shocks and $H^{h\prime}$ is a 3 by 2 matrix:

$$H^{h\prime} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{vmatrix}$$

The variances of the productivity and the signals are:

$$\tilde{P} = var_t\left(\xi_{t+1}\right) = \begin{vmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_a^2 \end{vmatrix} \qquad R^h = var_t\left(w_t^{jh}\right) = \begin{vmatrix} 2\lambda^2\theta\sigma_a^2 & 0 & 0 \\ 0 & \sigma_{H,H}^2 & 0 \\ 0 & 0 & \sigma_{H,F}^2 \end{vmatrix}$$

Based on her information, the Home agent's assessment of the expected productivity shocks and their variance are:

$$E_t^{jh}\left(\xi_{t+1}\right) = M^h Y_t^{jh} \qquad\qquad Var_t^{jh}\left(\xi_{t+1}\right) = \tilde{P} - M^h H^{h\prime} \tilde{P}$$

where $M^h$ is a 2 by 3 matrix:

$$M^h = \tilde{P} H^h \left[H^{h\prime} \tilde{P} H^h + R^h\right]^{-1}$$

**Expected productivity shocks**

The expected values of future Home and Foreign productivities are:

$$\begin{aligned} E_t^{jh}\varepsilon_{H,t+1} &= \alpha_x^{h,H} x_t^D + \alpha_{vH}^{h,H} v_{j,t}^{H,H} + \alpha_{vF}^{h,H} v_{j,t}^{H,F} \\ E_t^{jh}\varepsilon_{F,t+1} &= \alpha_x^{h,F} x_t^D + \alpha_{vH}^{h,F} v_{j,t}^{H,H} + \alpha_{vF}^{h,F} v_{j,t}^{H,F} \end{aligned}$$

where:

$$\alpha_x^{h,H} = \frac{\sigma_{H,H}^2 \left[\sigma_a^2 + \sigma_{H,F}^2\right]}{V}$$

$$\alpha_{vH}^{h,H} = \sigma_a^2 \frac{2\lambda^2\theta \left[\sigma_a^2 + \sigma_{H,F}^2\right] + \sigma_{H,F}^2}{V}$$

$$\alpha_{vF}^{h,H} = \frac{\sigma_a^2 \sigma_{H,H}^2}{V}$$

$$\alpha_x^{h,F} = -\frac{\sigma_{H,F}^2 \left[\sigma_a^2 + \sigma_{H,H}^2\right]}{V}$$

$$\alpha_{vH}^{h,F} = \frac{\sigma_a^2 \sigma_{H,F}^2}{V}$$

$$\alpha_{vF}^{h,F} = \sigma_a^2 \frac{2\lambda^2\theta \left[\sigma_a^2 + \sigma_{H,H}^2\right] + \sigma_{H,H}^2}{V}$$

$$V = 2\left[1 + \lambda^2\theta\right]\left[\sigma_a^2 + \sigma_{H,H}^2\right]\left[\sigma_a^2 + \sigma_{H,F}^2\right]$$
$$\quad - \sigma_a^2\left[\sigma_a^2 + \sigma_{H,H}^2\right] - \sigma_a^2\left[\sigma_a^2 + \sigma_{H,F}^2\right]$$

While these coefficients are complex functions, we can distinguish between their various orders. We consider components up to order two. The coefficients on $x_t^D$ ($\alpha_x^{h,H}$ and $\alpha_x^{h,F}$) only have components of order zero and two:

$$\alpha_x^{h,H}(0) = -\alpha_x^{h,F}(0) = \frac{1}{2\left[1 + \lambda^2\theta\right]}$$

$$\alpha_x^{h,H}(2) = \frac{\sigma_{H,H}^2 - \left[1 + 2\lambda^2\theta\right]\sigma_{H,F}^2}{4\left[1 + \lambda^2\theta\right]^2 \sigma_{H,H}^2 \sigma_{H,F}^2}\sigma_a^2$$

$$\alpha_x^{h,F}(2) = \frac{\left[1 + 2\lambda^2\theta\right]\sigma_{H,H}^2 - \sigma_{H,F}^2}{4\left[1 + \lambda^2\theta\right]^2 \sigma_{H,H}^2 \sigma_{H,F}^2}\sigma_a^2$$

The coefficients on the private signals only have components of order two:

$$\alpha_{vH}^{h,H}(2) = \frac{1 + 2\lambda^2\theta}{2\left[1 + \lambda^2\theta\right]\sigma_{H,H}^2}\sigma_a^2 \quad , \quad \alpha_{vF}^{h,H}(2) = \frac{1}{2\left[1 + \lambda^2\theta\right]\sigma_{H,F}^2}\sigma_a^2$$

$$\alpha_{vH}^{h,F}(2) = \frac{1}{2\left[1 + \lambda^2\theta\right]\sigma_{H,H}^2}\sigma_a^2 \quad , \quad \alpha_{vF}^{h,F}(2) = \frac{1 + 2\lambda^2\theta}{2\left[1 + \lambda^2\theta\right]\sigma_{H,F}^2}\sigma_a^2$$

The various orders of the Home agent's expectations of future Home produc-

tivity are then:

$$
\begin{aligned}
\left[E_t^{jh}\varepsilon_{H,t+1}\right](1) &= -\left[E_t^{ih}\varepsilon_{F,t+1}\right](1) = \alpha_x^{h,H}(0)\,x_t^D(1) \\
\left[E_t^{jh}\varepsilon_{H,t+1}\right](2) &= \alpha_{vH}^{h,H}(2)\,v_{j,t}^{H,H}(0) + \alpha_{vF}^{h,H}(2)\,v_{j,t}^{H,F}(0) \\
&= \alpha_{vH}^{h,H}(2)\,\epsilon_{j,t}^{H,H} + \alpha_{vF}^{h,H}(2)\,\epsilon_{j,t}^{H,F} \\
\left[E_t^{jh}\varepsilon_{F,t+1}\right](2) &= \alpha_{vH}^{h,F}(2)\,\epsilon_{j,t}^{H,H} + \alpha_{vF}^{h,F}(2)\,\epsilon_{j,t}^{H,F}
\end{aligned}
$$

A useful result if the third-order component of the expected productivity difference:

$$
\begin{aligned}
&\left[E_t^{jh}\left(\varepsilon_{H,t+1} - \varepsilon_{F,t+1}\right)\right](3) \\
={}& \left[\alpha_x^{h,H}(2) - \alpha_x^{h,F}(2)\right]x_t^D(1) + \left[\alpha_{vH}^{h,H}(2) - \alpha_{vH}^{h,F}(2)\right]v_{j,t}^{H,H}(1) + \left[\alpha_{vF}^{h,H}(2) - \alpha_{vF}^{h,F}(2)\right]v_{j,t}^{H,F}(1) \\
={}& \frac{-\lambda^2\theta\sigma_a^2}{2\left[1+\lambda^2\theta\right]^2}\frac{\sigma_{H,H}^2 + \sigma_{H,F}^2}{\sigma_{H,H}^2\sigma_{H,F}^2}x_t^D(1) + \frac{\lambda^2\theta\sigma_a^2}{1+\lambda^2\theta}\left(\frac{\varepsilon_{H,t+1}}{\sigma_{H,H}^2} - \frac{\varepsilon_{F,t+1}}{\sigma_{H,F}^2}\right) \quad (55)
\end{aligned}
$$

**Variance of productivity shocks**

The Home agent also infers the variances and covariances of the productivities shocks:

$$
\begin{aligned}
Var_t^{jh}\left(\varepsilon_{H,t+1}\right) &= \frac{\sigma_a^2\sigma_{H,H}^2}{V}\left[2\lambda^2\theta\left[\sigma_a^2 + \sigma_{H,F}^2\right] + \sigma_{H,F}^2\right] \\
Var_t^{jh}\left(\varepsilon_{F,t+1}\right) &= \frac{\sigma_a^2\sigma_{H,F}^2}{V}\left[2\lambda^2\theta\left[\sigma_a^2 + \sigma_{H,H}^2\right] + \sigma_{H,H}^2\right] \\
Covar_t^{jh}\left(\varepsilon\varepsilon_{H,t+1}, \varepsilon_{F,t+1}\right) &= \frac{\sigma_a^2\sigma_{H,H}^2\sigma_{H,F}^2}{V}
\end{aligned}
$$

These terms only have second-order components:

$$
\begin{aligned}
Var_t^{jh}\left(\varepsilon_{H,t+1}\right)(2) &= Var_t^h\left(\varepsilon_{F,t+1}\right)(2) = \frac{1+2\lambda^2\theta}{2\left[1+\lambda^2\theta\right]}\sigma_a^2 \\
Covar_t^{jh}\left(\varepsilon_{H,t+1}, \varepsilon_{F,t+1}\right)(2) &= \frac{1}{2\left[1+\lambda^2\theta\right]}\sigma_a^2
\end{aligned}
$$

The expected values of squared and cubic shocks are computed as:

$$
\begin{aligned}
E_t^{jh}\left(\varepsilon_{H,t+1}\right)^2 &= \left(E_t^{jh}\varepsilon_{H,t+1}\right)^2 + Var_t^{jh}\left(\varepsilon_{H,t+1}\right) \\
E_t^{jh}\left(\varepsilon_{H,t+1}\right)^3 &= \left(E_t^{jh}\varepsilon_{H,t+1}\right)^3 + 3\left(E_t^{jh}\varepsilon_{H,t+1}\right)\left(Var_t^{jh}\left(\varepsilon_{H,t+1}\right)\right) \\
E_t^{jh}\left(\varepsilon_{F,t+1}\right)^2 &= \left(E_t^{jh}\varepsilon_{F,t+1}\right)^2 + Var_t^{jh}\left(\varepsilon_{F,t+1}\right) \\
E_t^{jh}\left(\varepsilon_{F,t+1}\right)^3 &= \left(E_t^{jh}\varepsilon_{F,t+1}\right)^3 + 3\left(E_t^{jh}\varepsilon_{F,t+1}\right)\left(Var_t^{jh}\left(\varepsilon_{F,t+1}\right)\right)
\end{aligned}
$$

# C First order solution

To a first order, the variables are linear functions of the state space:

$$
\begin{aligned}
q_t^D\left(1\right) &= \alpha\left(0\right) S_t\left(1\right) + \alpha_5\left(0\right) x_t^D\left(1\right) \\
c_{yt}^A\left(1\right) &= \alpha_y\left(0\right) S_t\left(1\right) + \alpha_{5y}\left(0\right) x_t^D\left(1\right) \\
c_{yt}^D\left(1\right) &= \alpha_{yD}\left(0\right) S_t\left(1\right) + \alpha_{5,yD}\left(0\right) x_t^D\left(1\right) \\
q_t^A\left(1\right) &= \alpha_{qA}\left(0\right) S_t\left(1\right) + \alpha_{5,qA}\left(0\right) x_t^D\left(1\right) \\
k_{t+1}^A\left(1\right) &= \alpha_{kA}\left(0\right) S_t\left(1\right) + \alpha_{5,kA}\left(0\right) x_t^D\left(1\right) \\
k_{t+1}^D\left(1\right) &= \alpha_{kD}\left(0\right) S_t\left(1\right) + \alpha_{5,kD}\left(0\right) x_t^D\left(1\right) \\
z_{t+1}^A\left(1\right) &= \alpha_{zA}\left(0\right) S_t\left(1\right) + \alpha_{5,zA}\left(0\right) x_t^D\left(1\right)
\end{aligned}
$$

where:

$$
\begin{aligned}
S_t\left(1\right) &= \left[a_t^D\left(1\right), a_t^A\left(1\right), k_t^D\left(1\right), k_t^A\left(1\right)\right]' \\
x_t^D\left(1\right) &= \varepsilon_{t+1}^D + \lambda\left(\tau_t^D/\tau\right) \tag{56}
\end{aligned}
$$

**Worldwide averages**

The solution in terms for the worldwide averages of consumption, equity prices and capital dynamics is computed by taking first-order expansions of the equations (43)-(54), and take worldwide averages of the relations for the Home and Foreign country. The complete solution is given by:

$$
c_{yt}^A(1) = \lambda_1 a_t^A(1) + \lambda_2 k_t^A(1) \tag{57}
$$

$$
q_t^A(1) = \frac{\xi}{1+\xi}\frac{1-\bar{c}\lambda_1}{1-\bar{c}}a_t^A(1) - \frac{\xi}{1+\xi}\frac{\bar{c}\left(\lambda_2-1\right)+\omega}{1-\bar{c}}k_t^A(1) \tag{58}
$$

$$
k_{t+1}^A(1) = \frac{1}{1+\xi}\frac{1-\bar{c}\lambda_1}{1-\bar{c}}a_t^A(1) + \left(1 - \frac{1}{1+\xi}\frac{\bar{c}\left(\lambda_2-1\right)+\omega}{1-\bar{c}}\right)k_t^A(1) \tag{59}
$$

where:

$$
r_q = \frac{1-\delta}{\left(1-\omega\right)e^{-\omega k(0)} + \left(1-\delta\right)}
$$

$\lambda_1$ is given by:

$$
\begin{aligned}
&\lambda_1\left[1 + \bar{c}\frac{1-\gamma}{\gamma}\left[\frac{\xi}{1+\xi}\left(1-r_q\rho\right) + \left[r_q\frac{\xi}{1+\xi}\frac{\bar{c}\left(\lambda_2-1\right)+\omega}{1-\bar{c}} + \left(1-r_q\right)\omega\right]\frac{1}{1+\xi}\right]\right] \\
&= 1 + \frac{1-\gamma}{\gamma}\left[\frac{\xi}{1+\xi}\left(1-r_q\rho\right) + \left[r_q\frac{\xi}{1+\xi}\frac{\bar{c}\left(\lambda_2-1\right)+\omega}{1-\bar{c}} + \left(1-r_q\right)\omega\right]\frac{1}{1+\xi}\right] \\
&\quad - \frac{\left(1-\gamma\right)\left(1-\bar{c}\right)}{\gamma}\left(1-r_q\right)\rho
\end{aligned}
$$

$\lambda_2$ is given by:

$$\lambda_2 = 1 + \frac{(1 + \xi)(1 - \bar{c})\Phi - \omega}{\bar{c}}$$

where $\Phi \in [0, 1]$ is the coefficient on $k_t^A(1)$ in (59) and is the root of the polynomial:

$$0 = \Phi \left[ 1 + \xi + \bar{c}\frac{1 - \gamma}{\gamma}(1 - r_q)(\xi + \omega) \right] - \frac{1}{\gamma} \left[ \gamma \left[ 1 - \bar{c}(1 - r_q) \right] + \bar{c}(1 - r_q) \right] \omega + \bar{c}\frac{1 - \gamma}{\gamma} r_q \xi (\Phi)^2$$

**Cross-country differences**

The solution is relies on taking first-order expansions of the equations (43)-(54), and express them in terms of cross-country differences. The results are:

$$q_t^D(1) = \alpha_1(0) a_t^D(1) + \alpha_3(0) k_t^D(1) + \alpha_5(0) x_t^D(1) \tag{60}$$

$$c_{yt}^D(1) = a_t^D(1) + (1 - \omega)k_t^D(1) \tag{61}$$

$$k_{t+1}^D(1) = \frac{\alpha_1(0)}{\xi} a_t^D(1) + \left( 1 + \frac{\alpha_3(0)}{\xi} \right) k_t^D(1) + \frac{\alpha_5(0)}{\xi} x_t^D(1) \tag{62}$$

$$4z_t^A(1) = \left( \frac{1 + \xi}{\xi} \alpha_1(0) - z^D(0) \right) a_t^D(1) \tag{63}$$

$$+ \left( 1 + \frac{1 + \xi}{\xi} \alpha_3(0) - (1 - \omega)z^D(0) \right) k_t^D(1) + \frac{1 + \xi}{\xi} \alpha_5(0) x_t^D(1)$$

where:

$$\alpha_3(0) = \frac{1}{2r_q} \left[ (1 - r_q)(\xi + \omega) - \left( (1 - r_q)^2 (\xi + \omega)^2 + 4r_q\omega(1 - r_q)\xi \right)^{0.5} \right]$$

$$\alpha_1(0) = \frac{(1 - r_q)\rho}{1 + [\omega(1 - r_q) - r_q\alpha_3(0)]\frac{1}{\xi} - r_q\rho}$$

$$\alpha_5(0) = \frac{1 - r_q + r_q\alpha_1(0)}{1 + [\omega(1 - r_q) - r_q\alpha_3(0)]\frac{1}{\xi}} \frac{1}{1 + \lambda^2\theta}$$

and $\theta$ is the ratio between the variance of liquidity and productivity shocks: $\sigma_\tau^2 = \theta\sigma_a^2$. The coefficient $\alpha_5(0)$ in (60)-(63) is defined conditional on the term $\lambda$ in (56). In the absence of signal extraction $\alpha_5(0) = 0$ and the first-order cross-country solution is given by (60)-(63).

To solve for $\lambda$, we first take the third-order component of the optimal portfolio

condition for a Home investor (47) which can be written as:

$$
\begin{aligned}
& \gamma z_{Hj,t}\left(1\right)\left[E_t^{Hj}\left(er_{t+1}\right)^2\right](2) \\
= \; & \left[E_t^{Hj}er_{t+1}\right](3) + \tau_{Hj,t}\left(3\right) + (1-\gamma)\left[E_t^{Hj}er_{t+1}r_{t+1}^A\right](3) \\
& + (1-\gamma)\,\tau_{Hj,t}\left(2\right)\left[E_t^{Hj}r_{t+1}^A\right](1) \\
& - \left[\frac{1-\gamma}{2} + \gamma\left(2z_{Hj}\left(0\right)-1\right)\right]\tau_{Hj,t}\left(2\right)\left[E_t^{Hj}er_{t+1}\right](1) \\
& - \gamma\frac{2z_{Hj}\left(0\right)-1}{2}\left[E_t^{Hj}\left(er_{t+1}\right)^2\right](3) \\
& + \left[-\frac{\gamma\left(1+\gamma\right)}{2}z_{Hj}\left(0\right)\left(1-z_{Hj}\left(0\right)\right) + \frac{1}{6} - \frac{1-\gamma}{2}\frac{1+\gamma}{4}\right]\left[E_t^{Hj}\left(er_{t+1}\right)^3\right](3) \\
& + \frac{\left(1-\gamma\right)^2}{2}\left[E_t^{Hj}\left[r_{t+1}^A\right]^2 er_{t+1}\right](3) \\
& - \gamma\left(1-\gamma\right)\frac{2z_{Hj}\left(0\right)-1}{2}\left[E_t^{Hj}r_{t+1}^A\left(er_{t+1}\right)^2\right](3)
\end{aligned}
$$

We can undertake similar steps for the optimal portfolio condition for a Foreign investor (48). We then sum across investors to get a relation in terms of per capita variables in each country. Taking the average of these relations in the Home and the Foreign country, we get:

$$
\begin{aligned}
& 2\gamma z_t^A\left(1\right)\left[E_t\left(er_{t+1}\right)^2\right](2) \\
= \; & \int\left[E_t^{Hj}er_{t+1}\right](3)\,dj + \int\left[E_t^{Fj}er_{t+1}\right](3)\,dj + \tau_t^D(3) \\
& + (1-\gamma)\left(\int\left[E_t^{Hj}er_{t+1}r_{t+1}^A\right](3)dj + \int\left[E_t^{Fj}er_{t+1}r_{t+1}^A\right](3)dj\right) \\
& + \frac{\left(1-\gamma\right)^2}{2}\left(\int\left[E_t^{Hj}\left[r_{t+1}^A\right]^2 er_{t+1}\right](3)dj + \int\left[E_t^{Fj}\left[r_{t+1}^A\right]^2 er_{t+1}\right](3)dj\right) \quad (64) \\
& + (1-\gamma)\,\tau(2)\left[\int\left(E_t^{Hj}r_{t+1}^A\right)(1)\,dj - \int\left(E_t^{Hj}r_{t+1}^A\right)(1)\,dj\right] \\
& - \gamma\left(1-\gamma\right)\frac{z^D(0)}{2}\left[\int\left(E_t^{Hj}r_{t+1}^A(er_{t+1})^2\right)(3)dj - \int\left(E_t^{Fj}r_{t+1}^A(er_{t+1})^2\right)(3)dj\right] \\
& - \gamma\frac{z^D(0)}{2}\left[\int\left[E_t^{Hj}\left(er_{t+1}\right)^2\right](3)\,dj - \int\left[E_t^{Fj}\left(er_{t+1}\right)^2\right](3)\,dj\right]
\end{aligned}
$$

We can infer $\lambda$ from using (63) to substitute for $z_t^A\left(1\right)$ in (64). $x_t^D\left(1\right)$ enter several components of (64), and. $\tau_t^D(3)$ enters the second row of (64). Because agents do not observe the components of $x_t^D\left(1\right)$ separately, the model requires that

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$\varepsilon_{t+1}^D$ also enters (64) and that it does so in a way that when combined with $\tau_t^D(3)$ is enters as $x_t^D(1)$.

$\varepsilon_{t+1}^D$ does not enter through terms that are expectations of cross-products (as in lines 3 and following), as such terms would only lead to variances of shocks, or the expectation of $\varepsilon_{t+1}^D$. Instead $\varepsilon_{t+1}^D$ only enters (64) independently through the first-order component of the private signals, as this component are the actual shocks to future productivity. The signal extraction section above shows that the coefficients on private signals only have second-order components. The product of these coefficients and $\varepsilon_{t+1}^D$ is then of order three. $\varepsilon_{t+1}^D$ can then only enter (64) through a linear term, with the only such terms being:

$$\int \left[ E_t^{Hj} er_{t+1} \right] (3) \, dj + \int \left[ E_t^{Fj} er_{t+1} \right] (3) \, dj$$

To assess these terms, we can focus on a linear approximation of (51)-(54) :

$$er_{t+1} = -q_t^D + r_q q_{t+1}^D + (1 - r_q) \left( a_{t+1}^D - \omega k_{t+1}^D \right)$$

We can show that the only relevant terms in the expectation of $er_{t+1}$ for a Home investor are:

$$\left[ E_t^{Hj} er_{t+1} \right] (3) = \left[ r_q \alpha_1 (0) + (1 - r_q) \right] \left[ E_t^{ih} \left( \varepsilon_{H,t+1} - \varepsilon_{F,t+1} \right) \right] (3)$$

where $\left[ E_t^{ih} \left( \varepsilon_{H,t+1} - \varepsilon_{F,t+1} \right) \right] (3)$ is given by (55). We can undertake similar steps for a Foreign investors. Aggregating across individual investors, we obtain:

$$\int \left[ E_t^{Hj} er_{t+1} \right] (3) \, dj + \int \left[ E_t^{Fj} er_{t+1} \right] (3) \, dj$$

$$= \left[ r_q \alpha_1 (0) + (1 - r_q) \right] \frac{\lambda^2 \theta \sigma_a^2}{1 + \lambda^2 \theta} \left( \frac{1}{\sigma_{H,H}^2} + \frac{1}{\sigma_{H,F}^2} \right) \varepsilon_{t+1}^D$$

Focusing on the terms of interest, (64) becomes:

$$0 = \int \left[ E_t^{Hj} er_{t+1} \right] (3) \, dj + \int \left[ E_t^{Fj} er_{t+1} \right] (3) \, dj + \tau_t^D(3)$$

$$= \left[ r_q \alpha_1 (0) + (1 - r_q) \right] \sigma_a^2 \frac{\lambda^2 \theta}{1 + \lambda^2 \theta} \left( \frac{1}{\sigma_{H,H}^2} + \frac{1}{\sigma_{H,F}^2} \right) \varepsilon_{t+1}^D + \tau_t^D(3)$$

The ratio between the coefficient on $\varepsilon_{t+1}^D$ and the coefficient on $\tau_t^D(3)$ must be the same as in $x_t^D(1)$, implying

$$\left[ r_q \alpha_1 (0) + (1 - r_q) \right] \left( \frac{1}{\sigma_{H,H}^2} + \frac{1}{\sigma_{H,F}^2} \right) \lambda = \frac{1 + \lambda^2 \theta}{\lambda^2 \theta} \frac{\tau(2)}{\sigma_a^2} \qquad (65)$$

The left-hand side of (65) is an increasing linear function of $\lambda$ which is flatter the higher the variance of private signal. The right-hand side of (65) is decreasing function of $\lambda$ that is infinite when $\lambda \to 0$ and converges to $\tau(2)/\sigma_a^2$ when $\lambda \to \infty$. (65) therefore gives an implicit solution for $\lambda$. Combining it with (60)-(63) gives the first-order solution for the model.

# D   Portfolio difference

**Zero order solution**

We solve for $z^D(0) = z_H(0) - z_F(0)$ by taking the second-order component of the optimal portfolio condition for a Home investor (47) which can be written as:

$$z_{Hj}(0) = \frac{1}{2} + \frac{\left[E_t^{Hj} er_{t+1}\right](2) + \tau_{Hj,t}(2)}{\gamma \left[E_t^{Hj}(er_{t+1})^2\right](2)} + \frac{1-\gamma}{\gamma} \frac{\left[E_t^{Hj} er_{t+1} r_{t+1}^A\right](2)}{\left[E_t^{Hj}(er_{t+1})^2\right](2)}$$

We can undertake similar steps for the optimal portfolio condition for a Foreign investor (48). We then sum across investors to get a relation in terms of per capita variables in each country. Taking the difference between these relations in the Home and the Foreign country, we get:

$$z^D(0) = \frac{2\tau(2)}{\gamma \left[E_t(er_{t+1})^2\right](2)} + \frac{\int \left[E_t^{Hj} er_{t+1}\right](2)\, dj - \int \left[E_t^{Fj} er_{t+1}\right](2)\, dj}{\gamma \left[E_t(er_{t+1})^2\right](2)}$$

$$+(1-\gamma)\frac{\int \left[E_t^{Hj} er_{t+1} r_{t+1}^A\right](2)\, dj - \int \left[E_t^{Fj} er_{t+1} r_{t+1}^A\right](2)\, dj}{\gamma \left[E_t(er_{t+1})^2\right](2)}$$

We can show that $\left[E_t^{Hj} er_{t+1} r_{t+1}^A\right](2) = \left[E_t^{Fj} er_{t+1} r_{t+1}^A\right](2) = 0$ and $\left[E_t^{Hj} er_{t+1}\right](2) = \left[E_t^{Fj} er_{t+1}\right](2)$. In addition:

$$\left[E_t^{Hj}(er_{t+1})^2\right](2) = 2\left[(1 - r_q + r_q \alpha_1(0))^2 \frac{\lambda^2 \theta}{1 + \lambda^2 \theta} + (r_q \alpha_5(0))^2 \left[1 + \lambda^2 \theta\right]\right] \sigma_a^2$$

$$= 2\sigma_a^2(1 - r_q + r_q \alpha_1(0))^2 \Gamma$$

where $\Gamma \in [0,1]$ is an increasing function of $\lambda$ that converges to one when private signals are infinitely noisy ($\lambda \to \infty$):

$$\Gamma = 1 - \left(1 - \left(\frac{r_q}{1 + [\omega(1 - r_q) - r_q \alpha_3(0)]\frac{1}{\xi}}\right)\right)^2 \frac{1}{1 + \lambda^2 \theta}$$

The zero-order portfolio difference is then::

$$z^D(0) = \frac{2\tau(2)}{\gamma \left[E_t(er_{t+1})^2\right](2)} = \frac{\tau(2)}{\gamma\sigma_a^2} \frac{1}{(1 - r_q + r_q\alpha_1(0))^2} \frac{1}{\Gamma} \tag{66}$$

**First-order solution**

The first-order component of the difference in portfolio shares is solved by taking the third-order component of the optimal portfolio condition for a Home investor (47), and aggregating across Home investors to obtain a per capita average for the Home country. We follow similar steps with the third-order component of the optimal portfolio condition for a Foreign investor (48). Taking the difference between the Home and Foreign per-capita relations we write:

$$\gamma z_t^D(1)\left[E_t(er_{t+1})^2\right](2)$$

$$= \int \left[E_t^{Hj} er_{t+1}\right](3)\,dj - \int \left[E_t^{Fj} er_{t+1}\right](3)\,dj + 2\tau_t^A(3)$$

$$+ (1-\gamma)\left(\int \left[E_t^{Hj} er_{t+1} r_{t+1}^A\right](3)dj - \int \left[E_t^{Fj} er_{t+1} r_{t+1}^A\right](3)dj\right)$$

$$+ (1-\gamma)\tau(2)\left[\int \left(E_t^{Hj} r_{t+1}^A\right)(1)\,dj + \int \left(E_t^{Hj} r_{t+1}^A\right)(1)\,dj\right]$$

$$- \gamma\frac{2z_H(0) - 1}{2}\left(\int \left[E_t^{Hj}(er_{t+1})^2\right](3)\,dj + \int \left[E_t^{Fj}(er_{t+1})^2\right](3)\,dj\right)$$

$$+ \frac{(1-\gamma)^2}{2}\left(\int \left[E_t^{Hj}\left[r_{t+1}^A\right]^2 er_{t+1}\right](3)dj - \int \left[E_t^{Fj}\left[r_{t+1}^A\right]^2 er_{t+1}\right](3)dj\right)$$

$$- \gamma(1-\gamma)\frac{2z_H(0) - 1}{2}\left(\int \left(E_t^{Hj} r_{t+1}^A(er_{t+1})^2\right)(3)dj + \int \left(E_t^{Fj} r_{t+1}^A(er_{t+1})^2\right)(3)dj\right)$$

The various terms can be computed using the first- and second-order components of the solution. The detailed steps are complex and the solution takes the form:

$$z_t^D(1) = \frac{z^D(0)}{2\lambda}\frac{\sigma_{H,F}^2 - \sigma_{H,H}^2}{\sigma_{H,H}^2 + \sigma_{H,F}^2}\varepsilon_{t+1}^A(1) + z^D(0)\frac{\tau_t^A(3)}{\tau(2)} + \Omega_S S_t(1) + \frac{f\left(\sigma_a^2 x_t^D(1), \left[x_t^D(1)\right]^3\right)}{\left[E_t(er_{t+1})^2\right](2)}$$

where $\Omega_S$ is a zero-order parameter and $f$ is complex function.

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# E   Balance of Payments Accounting

**Saving and investment**

Saving is equal to income minus consumption. In line with national accounts, we consider savings net of the amount required to offset the depreciation of capital. National saving in the Home and Foreign countries are:

$$
\begin{aligned}
S_t^H &= \int \left( w_{H,t} - C_{y,t}^{jH} \right) dj \\
&\quad - \int \left[ z_{Hj,t-1} \frac{Q_{H,t}}{Q_{H,t-1}} + (1 - z_{Hj,t-1}) \frac{Q_{F,t}}{Q_{F,t-1}} \right] \left( w_{H,t-1} - C_{y,t-1}^{jH} \right) dj \\
S_t^F &= \int \left( w_{F,t} - C_{y,t}^{jF} \right) dj \\
&\quad - \int \left[ z_{Fj,t-1} \frac{Q_{H,t}}{Q_{H,t-1}} + (1 - z_{Fj,t-1}) \frac{Q_{F,t}}{Q_{F,t-1}} \right] \left( w_{F,t-1} - C_{y,t-1}^{jF} \right) dj
\end{aligned}
$$

The first-order components of savings are:

$$
\begin{aligned}
s_t^H (1) &= \frac{1}{1 - \bar{c}} \left[ \Delta a_{H,t} (1) + (1 - \omega) \Delta k_{H,t} (1) \right] \\
&\quad - \frac{\bar{c}}{1 - \bar{c}} \Delta c_{y,t}^H (1) - \Delta q_t^A (1) - \frac{z^D (0)}{2} \Delta q_t^D (1) \\
s_t^F (1) &= \frac{1}{1 - \bar{c}} \left[ \Delta a_{F,t} (1) + (1 - \omega) \Delta k_{F,t} (1) \right] \\
&\quad - \frac{\bar{c}}{1 - \bar{c}} \Delta c_{y,t}^F (1) - \Delta q_t^A (1) + \frac{z^D (0)}{2} \Delta q_t^D (1)
\end{aligned}
$$

where $s_t^i (1)$ is the first-order component of savings, scaled by the steady state wealth: $s_t^i (1) = S_t^i (1) / (W (0) (1 - \bar{c}))$. In addition for a variable $g$: $\Delta g_t (1) = g_t (1) - g_{t-1} (1)$. The first-order consumption in a country can be split between the world-wide average and the cross-country difference. Using (57) and (61), consumption in a specific country reflects only the observed state variables:

$$
\begin{aligned}
\Delta c_{y,t}^H (1) &= \Delta c_{y,t}^A (1) + \frac{1}{2} \Delta c_{y,t}^D (1) \\
&= \lambda_1 \Delta a_t^A (1) + \lambda_2 \Delta k_t^A (1) + \frac{1}{2} \left[ \Delta a_t^D (1) + (1 - \omega) \Delta k_t^D (1) \right]
\end{aligned}
$$

Saving in a specific country are then affected by the information dispersion only

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through relative equity prices:

$$s_t^H(1) = \alpha_H \Delta S_t(1) - \frac{z^D(0)}{2} \Delta q_t^D(1)$$

$$s_t^F(1) = \alpha_F \Delta S_t(1) - \frac{z^D(0)}{2} \Delta q_t^D(1)$$

$$s_t^D(1) = \Delta a_t^D(1) + (1-\omega)\Delta k_t^D(1) - z^D(0)\Delta q_t^D(1)$$

where $s_t^D(1) = s_t^H(1) - s_t^F(1)$.

Investment is also defined net of depreciation:

$$I_{i,t}^{net} = I_{i,t} - \delta K_{i,t-1} = K_{i,t} - K_{i,t-1} \qquad\qquad i = H, F$$

The first-order component of investment, scaled by steady-state wealth, is then:

$$i_t^{D,net}(1) = \frac{I_t^{H,net}(1) - I_t^{F,net}(1)}{e^{w(0)}(1-\bar{c})} = \Delta k_{t+1}^D(1) = \frac{1}{\xi} q_t^D(1)$$

where we used (5).

**Capital flows**

The passive portfolio share combines the steady-state holdings of quantities of assets with the actual asset prices. For Home investors, we write:

$$z_{H,t}^p = \frac{z_H(0) e^{q_{H,t}}}{z_H(0) e^{q_{H,t}} + (1 - z_H(0)) e^{q_{F,t}}}$$

The first-order passive portfolio share is the same for all investors:

$$z_t^p(1) = z_H(0)(1 - z_H(0)) q_t^D(1)$$

Using the difference between the first-order components of (49) and (50) we get:

$$\Delta z_t^A(1) - \Delta z_t^p(1) = \frac{1}{4}\left[ i_t^{D,net}(1) - z^D(0) s_t^D(1) \right]$$

Gross capital outflows and inflows reflect the changes in the value of cross-border asset holdings:

$$OUTFLOWS_t = \int (1 - z_{Hj,t})\left(w_{H,t} - C_{y,t}^{jH}\right) dj$$
$$- \frac{Q_{F,t}}{Q_{F,t-1}} \int (1 - z_{Hj,t-1})\left(w_{H,t-1} - C_{y,t-1}^{jH}\right) dj$$

$$INFLOWS_t = \int z_{Fj,t}\left(w_{F,t} - C_{y,t}^{jF}\right) dj$$
$$- \frac{Q_{H,t}}{Q_{H,t-1}} \int z_{Fj,t-1}\left(w_{F,t-1} - C_{y,t-1}^{jF}\right) dj$$

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The first-order components, scaled by steady-state wealth $W(0)(1 - \bar{c})$, are:

$$
\begin{aligned}
outflows_t &= -\left(1 - z_H(0)\right)\Delta q_{F,t}(1) - \Delta z_{H,t}(1) \\
&\quad + \frac{1 - z_H(0)}{1 - \bar{c}}\left[\Delta a_{H,t}(1) + (1 - \omega)\Delta k_{H,t}(1) - \bar{c}\Delta c_{y,t}^{H}(1)\right] \\
&= \left(1 - z_H(0)\right)s_t^{H}(1) - \left[\Delta z_{H,t}(1) - \Delta z_t^{p}(1)\right] \\
inflows_t &= \left(1 - z_H(0)\right)s_t^{F}(1) + \left[\Delta z_{F,t}(1) - \Delta z_t^{p}(1)\right]
\end{aligned}
$$

In terms of net capital flows, we write:

$$
\begin{aligned}
net_t &= outflows_t - inflows_t \\
&= \left(1 - z_H(0)\right)s_t^{D}(1) - 2\left[\Delta z_t^{A}(1) - \Delta z_t^{p}(1)\right] \\
&= \frac{1}{2}\left[s_t^{D}(1) - i_t^{D,net}(1)\right]
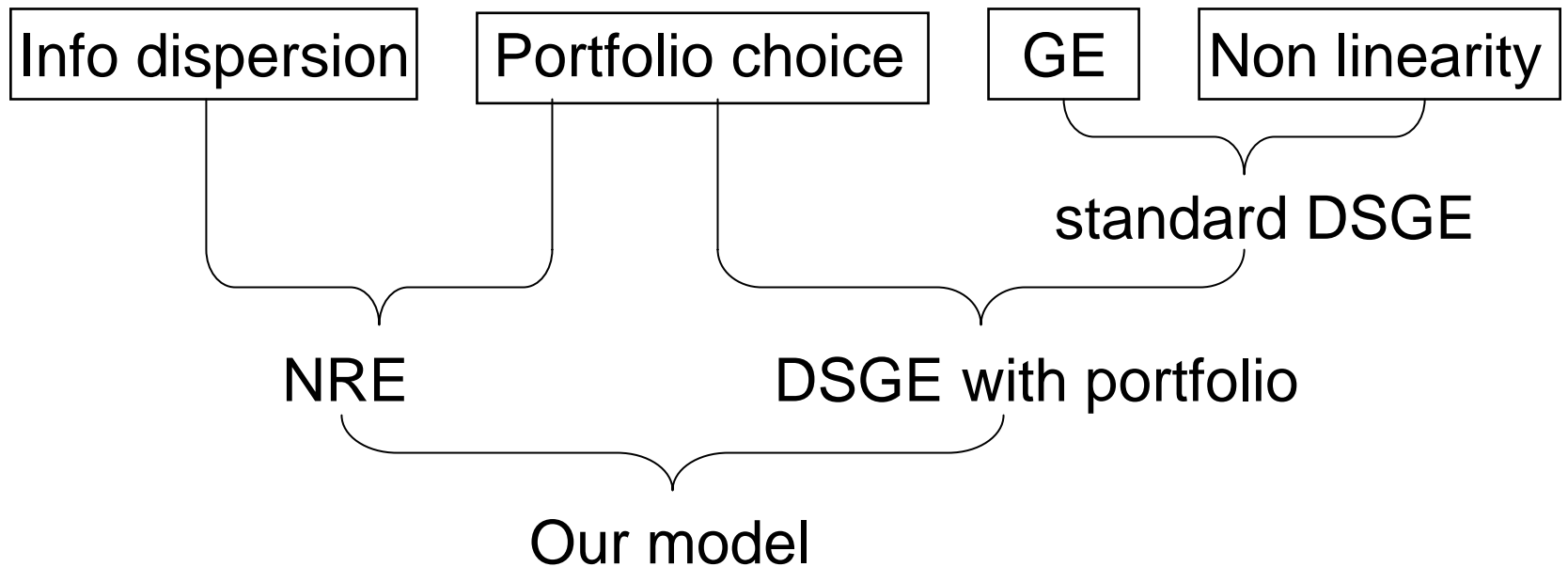\end{aligned}
$$

The sum of gross capital flows is:

$$
outflows_t + inflows_t = \frac{1 - z_H(0)}{2}s_t^{A}(1) - \Delta z_t^{D}(1)
$$

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# Figure 1 Modeling Contribution



Info dispersion    Portfolio choice    GE    Non linearity

standard DSGE

NRE                DSGE with portfolio

Our model

# Figure 2  Role of Information Dispersion

$\tau_t^D$    $\varepsilon_{t+1}^D$    $\varepsilon_{t+1}^A$

$x_t^D$

$[\overline{E}_{H,t} er_{t+1}](3) - [\overline{E}_{F,t} er_{t+1}](3)$

$q_t^D$    $[\text{var}_t er_{t+1}](3)$

saving, investment

$[\overline{E}_t^A er_{t+1}]^{IS}(3)$

portfolio growth

Capital flows