The cost of financial distress and the timing of default *

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Abstract

At any point in time, most firms are not in financial distress. This implies that they must suffer value losses unrelated to their leverage - economic shocks - before becoming financially distressed. We show that if estimates of ex-ante financial distress costs are not filtered from the effects of future economic shocks, they are biased upward by a factor of five on average, and up to an order of magnitude. Filtered from economic shocks, pure ex-ante distress costs average less than 1% of current firm value. We also estimate sensitivities of ex-ante distress costs to leverage that are generally far too small to offset the expected tax benefits. Extending our analysis to the cross-section and time series, we confirm that ex-ante distress costs are highest: i) when the risk premium in debt markets is high, and ii) among firms with high systematic risk. Overall, our results suggest that most firms use debt too conservatively, but we characterize conditions under which they do not.

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1 Introduction

The textbook version of the trade-off theory posits that debt financing is preferred to equity as long as each new dollar of debt creates more in tax shields than financial distress or bankruptcy costs. Yet, because each has proven hard to measure, it has until recently been difficult to evaluate the theory's ability to explain observed debt ratios. On the tax side, a series of papers by Graham argues that taxes matter, and a lot. Incorporating complexities of the tax code (e.g., net-operating-losses and investment credits), Graham (2000) quantifies the "true" marginal tax rate for a large sample of firms, showing that most could (at least) double their leverage before the tax benefits begin to abate.[1] For this finding to be consistent with the trade-off theory, financial distress costs must be quite sensitive to leverage, even at small debt ratios.

To examine this issue, several studies have attempted to characterize the present value of expected financial distress costs. The usual recipe has three ingredients: 1) forecast dollar losses given distress, 2) multiply this by the probability of becoming distressed, and 3) discount to present value. The first ingredient generally comes from studies of ex-post distress or default, such as Altman (1984), Opler and Titman (1994), or Andrade and Kaplan (1998). Although the exact figures differ across studies, it can be reasonably assumed that once a firm becomes distressed, it can expect to lose an additional 10% - 25% due to investment distortions, predation by well-financed competitors, inability to maintain relationships with customers, etc.

Capitalizing these ex-post losses into ex-ante values requires the second and third ingredients, each of which have been the subject of recent work. Molina (2005) uses changes in a firm's credit rating to measure changes in its distress probability and finds that the impact of leverage on distress probabilities, and by extension the present value of financial distress costs, can be sensitive enough to offset Graham's (2000) potential tax savings.[2] Almeida and Philippon (2007) take this a step further, arguing that because distress risk is systematic (Ross (1985)), using *historical* default probabilities to price ex-ante financial distress will undershoot its present value. Extracting *risk-adjusted* probabilities from credit spreads, they present evidence suggesting that tax benefits and risk-adjusted financial distress costs can have similar sensitivities to leverage, providing further support for the trade-off theory.

[1] As firms add more leverage (holding profitability constant), taxable income declines, which reduces marginal tax rates. Graham (2000) estimates the leverage "kink" where these benefits start to decline, and comparing observed leverage ratios to this kink, argues that substantial money is left on the table via suboptimally low leverage.

[2] A key insight in his paper is that a firm's credit rating depends not only on its leverage, but also on other firm characteristics that alter its likelihood of distress (e.g., operating risk, profitability, etc.). He develops an econometric technique which accounts for the ensuing endogeneity.

This paper builds upon these insights by emphasizing an additional issue that, if ignored, will bias estimates of ex-ante financial distress costs upward by a factor of five on average, and up to an order of magnitude. The issue is distinguishing between downward economic shocks (that have nothing to do with the firm's financing) from financial distress costs (that have everything to do with it). The importance of this distinction has long been recognized in ex-post studies such as Asquith et al. (1994), Opler and Titman (1994), and Andrade and Kaplan (1998), but in ex-ante calculations has not been similarly appreciated. In this paper, we implement an approach that makes this ex-ante distinction, and then analyze whether the key prediction of the trade-off theory still holds, i.e., whether ex-ante tax benefits and financial distress costs have similar sensitivities to leverage. In the vast majority of cases, we find that they do not.

To appreciate how much ex-ante financial distress costs are biased without netting out expected economic shocks, consider a hypothetical firm that can only experience distress at some future date t. Assume that the risk-adjusted probability of experiencing distress is 10%, and that conditional on distress, the firm expects to lose 23% of its time t value to financial distress (the upper end of Andrade and Kaplan's range). With a zero risk-free rate, multiplying these figures gives a present value of financial distress costs of 2.3%. However, this calculation overstates ex-ante financial distress costs, because it does not properly filter out economic shocks that would lead the firm to become distressed. For example, if the firm can suffer a 70% economic decline before becoming distressed (i.e., it will begin incurring financial distress costs at \$30), then the estimated ex-ante financial distress costs are too high by over a factor of three (2.3% vs. (10%)(23%)(\$100 - \$70)=\$.69, or .69% of current firm value). Sensitivities to leverage will be similarly inflated.

As this example illustrates, accounting for economic shocks in ex-ante financial distress calculations can make a substantial difference. This is also true in aggregate. Frank and Goyal (2007) find that the average ratio of total debt to market value of assets from 1950-2003 is around 0.29, such that at any point in time, a firm would need to lose over 70% of its value before defaulting. But even this may overstate the economic losses required for firms to experience financial distress. Davydenko (2007) examines the market values of firms at default, and finds that on average, they are around 60% of face debt obligations. Such a typical case would imply that the required shock to trigger default is about 82% ($100\% - 30\% \cdot 60\%$). This value, some 18% of the firm's value when the ex-ante distress calculation is made, corresponds to the benchmark used by ex-post studies such as Andrade and Kaplan (1998), and allows for a "pure" calculation of financial distress costs.

Calculating ex-ante distress costs firm by firm thus requires a model for values at or near default, so that future economic shocks can be filtered out. A natural approach is to use a structural model of default that provide both risk-adjusted default probabilities and values at default within a unified framework.[3] In this paper, we use the framework of Leland and Toft (1996), hereafter LT.[4] Not only is LT tractable and easily estimated, but also fits Davydenko's (2007) empirically estimated average values at default quite well. This benefit notwithstanding, neither our main point nor results are sensitive to the default rule specified by the LT model. We perform a number of robustness checks not only within the LT model, but also re-estimate everything with a different default specification altogether (Merton (1974)). Neither alters our main conclusions. This is unsurprising given that the differences in default boundaries across specifications is inconsequential, relative to not modeling default values at all.

The basic mechanics of the LT model are standard. The firm has two sources of value: 1) unlevered assets whose value can be described by a random process, and 2) debt that generates a tax shield against the firm's profits.[5] Normally, the fluctuation in firm value is due purely to shocks that have nothing to do with capital structure, i.e., economic shocks. Only when value drops below a critical threshold is bankruptcy is triggered. Should this occur, debt holders bear additional losses in proportion to the firm's value at that time. Importantly, this structure separates value declines that take a firm closer to bankruptcy (downward economic shocks) from those that are due to default itself (bankruptcy/financial distress costs).

To calculate ex-ante distress costs, we are ultimately after a sequence of risk-adjusted probabilities and corresponding default thresholds. One appealing and realistic feature of the LT model is that default is endogenous. At every point in the firm's life, equity holders decide whether to continue to service debt, even to the point of injecting additional equity to cover profit shortfalls. From this trade-off arises an endogenous default threshold that depends on a number of structural parameters. This threshold is useful in two dimensions. First, once we have inferred the asset volatility from estimating the model, we can infer the risk-adjusted default probability for any arbitrary future time. Second, estimates of *proportional* ex-post financial distress costs such as Andrade and Kaplan (1998) can be converted to ex-ante *dollar* losses, which are ultimately what firms care about when selecting their leverage.

To specify the default threshold, several structural parameters must be estimated. Some, such as interest expenses, payout rates, and total liabilities are drawn from standard data sources. Asset volatility is es-

[3] A partial list of these includes Merton (1974), Black and Cox (1976), Kim, Ramaswamy, and Sundaresan (1993), Leland (1994), and Longstaff and Schwartz (1995).

[4] The basic structure is similar to Leland (1994), with the exception that debt has a finite maturity.

[5] Like other standard debt pricing models such as Merton (1974), Black and Cox (1976), and Brennan and Schwartz (1978), Leland and Toft (1996) assume that the value of the firm's unlevered assets evolves according to Geometric Brownian dynamics.

timated by asking our model to explain observed equity data. compute a sequence of risk-neutral default probabilities and default thresholds for over 450,000 firm-quarter observations from 1980 to 2007. Every publicly traded firm with sufficient data to conduct the estimation is included in the analysis.

Our results are as follows. In aggregate, our risk-adjusted default probabilities match quite closely those reported by Almeida and Philippon (2007), and our default thresholds conform with Davydenko's (2007) empirical estimates. However, when we calculate ex-ante financial distress costs, we find that for most firms, they are quite modest. Using Andrade and Kaplan's (1998) midpoint of 16.5% for ex-post proportional losses, we find that ex-ante distress costs for the average firm comprise less than 1% of current value. For less than a tenth of firms do financial distress costs exceed 2.5%. These results stand in marked contrast to recent estimates by Almeida and Philippon (2007), which, also using the 16.5% midpoint, finds ex-ante distress costs in the neighborhood of 4-5%.[6] Importantly, the difference between their results and ours is *not* attributable to differences in risk-adjusted probabilities or different assumptions for ex-post loss rates in default. The risk-adjusted probabilities generated are very similar to theirs extracted from credit spreads, and we use identical estimates for proportional ex-post losses. Instead, our results are so much smaller because we distinguish between downward economic shocks that take firm's *closer* to default, from financial distress which kicks in *at* default.[7]

However, these results by themselves do not necessarily indicate that firms are underlevered. If the sensitivities of financial distress costs to leverage ratios are on par with the (absolute values of) sensitivities of tax benefits, then observed leverage ratios may in fact be close to the targets implied from the trade-off theory. Our approach to addressing this question offers three distinct advantages over existing ones. First, rather than inferring these from ratings changes, we can directly compute both the derivatives of ex-ante financial distress costs and tax benefits to leverage. This allows us to sidestep the endogeneity problems emphasized by Molina (2005). Secondly, our approach greatly expands the number of firms that can be analyzed, as a debt rating is not required to infer the likelihood of default. Finally, not only are financial distress costs adjusted for systematic risk (as in Almeida and Philippon (2007)), but so too are the tax benefits. The converse of default (more likely to occur in bad times) is survival (more likely to occur in good times), which implies that each should be adjusted for systematic risk.[8] We find that even when this risk adjustment is

[6] Their "benchmark" calculations, presented in Table IV, Panel A, p. 2571, are presented by credit rating: 9.54% (B), 6.81% (BB), 4.53% (BBB), 3.83% (A), 1.84% (AA), and 0.32% (AAA).

[7] When we then repeat our calculations for the Andrade and Kaplan endpoints of 10% and 23%, our findings are similar. Even at the upper extreme, it is uncommon for the present value of ex-ante financial distress costs to reach 1%.

[8] The LT makes this adjustment implicitly by capitalizing the tax benefits under the same risk-adjusted probability measure used to calculate financial distress costs.

made, in over 90% of our firm-quarter observations, firms would gain more in tax benefits than they would lose in distress costs. On average, an additional dollar of debt increases firm value by about 14 cents through the tax shield, but costs less than 5 cents in financial distress costs. This conclusion is robust to a wide variety of parameter assumptions about tax rates, proportional losses in default, and asset volatilities.

A additional advantage of estimating a structural model for a large panel of firms is that it affords insights into the time-series and cross-sectional patterns of ex-ante distress costs. Specifically, although our results suggest that ex-ante financial distress costs are generally too low to be reconciled with the static trade-off theory, this is not always the case. First, even in the absence of any risk-adjustment, the determinants of distress costs are volatile. For example, the average financial distress costs *without* risk premia (i.e., expected losses in distress under the objective probability measure) lie around 0.5% but vary significantly over time. In the early 1980s it lay at about 0.25%, shifted to about twice that level during the 1990s, and then increased almost by a factor three to reach a peak in 2002.

Second, because the risk premium associated with such losses also varies over time (see e.g., Berndt et al. (2008) and Ericsson and Elkamhi (2008)), the risk-adjusted cost of these losses exhibits even more time variation. The risk-adjusted distress costs vary between 0.25% to levels almost ten times higher, with a mean just less than 1%. The time series pattern is consistent with the variation in risk premia measured in credit markets: in times when the credit risk premium is high, financial distress costs are highest. Similar insight is found from examining the cross-section. For firms with the highest systematic risk (asset betas), the wedge between expected financial distress losses and the risk-adjusted price of these losses is highest. Finally, we compare the sensitivities of financial distress costs and tax benefits to leverage, both over time. We show that although on average the value of an additional dollar of debt offsets the marginal cost of financial distress more than a factor of two, at one point in our sample (late 2002), the two do in fact equate.

Our findings are directly relevant for a number of recent studies that have estimated ex-ante distress costs and/or their sensitivities to leverage. We demonstrate the importance of accounting for the values at which firms default. This permits estimates of ex-ante financial distress costs to filtered from the effects of future economic shocks. Ultimately, our analysis strongly suggests that for most firms, financial distress costs are insufficient to offset the substantial tax benefits documented by Graham (2000). These results also conform with survey evidence indicating that most managers do not (at least explicitly) consider financial distress particularly important. When asked to rate factors influencing leverage choices, financial distress ranked only seventh (Graham and Harvey (2001)).

Our work is also related to recent work by Van Binsbergen et al. (2008) and Korteweg (2008). The former also address the costs of financial distress and default. However they ask a different question - assuming firms are optimally leveraged, how large do distress costs have to be? The presence of substantial tax benefits (such as estimated in Graham (2000)) thus directly implies large losses due to financial distress or default. Our analysis differs by measuring default costs directly using risk-adjusted probabilities and empirical estimates from Andrade and Kaplan (1998), and in so doing, allows for (even the majority of) firms to be underleveraged.

Korteweg (2008) infers the net benefits of leverage for 269 firms by relying on constraints implied by Modigliani and Miller on firm values and their betas. His estimates for distress costs are higher than ours, in the neighborhood of 5% of firm value. This difference may underscore the difference between losses that occur at default, from those that occur prior.

However, there are clear limits to what we can conclude. Perhaps the most important (and unfortunately, the least observable) is one of timing - the extent to which financial distress begins prior to actual default. Here, we have little to add to the debate, other than to recognize (as many have before) that they are clearly not the same. While there is little doubt that even the threat of bankruptcy can lead to financial distress, including the loss of key employees (Sharpe (1994)), underinvestment (Myers (1977)), or predation (Bolton and Sharfstein (1990)), it is unclear how one would specify the probabilities of such pre-default distress.[9] In the absence of such a model, we follow Graham (2000), Molina (2005), and Almeida and Philippon (2007), and quantify ex-ante distress costs that are incurred when the firm defaults.

An additional limitation is that, like other studies of distress costs, our analysis of the trade-off theory considers only bankruptcy/distress costs versus tax shields, ignoring other potential effects of leverage including signaling, providing incentives (e.g., Jensen and Meckling (1976), Myers (1977), Parrino and Weisbach (1999)), or competition in the product market (e.g., Brander and Lewis (1986), Maksimovic (1986)).[10] There are clearly situations where one or more of these will be the dominant consideration for a firm's financing choices, in which case our specification is too simple. Richer structural specifications for financial distress with an eye on identifying the specific channels of financial distress would be helpful here, and would also lay the foundation for cross-sectional tests that could better evaluate the ability of structural models to measure ex-ante financial distress.

[9] See Tsyplakov and Titman (2007) for a structural model in which firms incur distress costs in advance of default.

[10] One might argue that the last of these represents another form of financial distress costs, which we are already capturing.

Immediately following, we describe how we calculate ex-ante distress costs in a way that filters out the effects of future economic shocks. Our results are presented in Section 3, including a discussion of both the *levels* of ex-ante financial distress costs as well as their *sensitivities* to leverage. In Section 4, we investigate the cross-sectional and time series patterns of distress costs, paying particular attention to the role of systematic risk. We then consider a number of robustness checks and extensions in Section 5, and then conclude.

2 Separating financial distress costs from economic shocks

A particularly challenging part about measuring financial distress is distinguishing it from performance declines that would have occurred even if bankruptcy were not the primary concern. We refer to these as downward economic shocks. A good illustration of an economic shock is how the credit crisis and ensuing recession of late 2008 adversely affected the automobile industry, whose members suffered substantial sales declines.[11] The fact that losses were so widespread regardless of financial health (e.g., Toyota's AA credit rating versus Ford's CCC) strongly suggests that the major players in this industry suffered a significant economic shock.

However, it is also quite likely that for the most highly leveraged firms, additional costs of financial distress costs may have compounded the effect of negative economic shocks. Examples include debt-equity holder conflicts, losing workers to financially healthy competitors, or stimulating rivals to cut prices. An online column by consumer website Kiplinger.com entitled, "Should you buy a car from Detroit now?" emphasizes one type of financial distress cost particularly relevant for automobile manufacturers, namely that concern for a firm's financial health can influence buying decisions (Titman, 1984).[12] Listing risks faced by consumers including whether warranties would be honored, whether parts would be available, and whether the car of a defunct manufacturer would quickly depreciate, the article asks simply, "Why take the chance?"

Given that economic shocks and financial distress costs can coexist, the empirical challenge then becomes to identify situations where they can be separately quantified. Ex-post, this is typically done by conditioning on value *at or near* distress, as Andrade and Kaplan (1998) do in their study of 31 firms who became distressed after highly leveraged transactions. We illustrate their timing with the help of Figure 1, which shows how a firm might evolve into and out of financial distress. On the y-axis is firm value, V_t, and on the x-axis, event

[11] Year-over-year sales in November 2008 for General Motors, Ford Motor Co., Chrysler, Toyota, Honda, and Nissan fell by 41.3%, 30.5%, 47.1%, 33.9%, 31.6%, and 42.2% respectively compared to the previous year.

[12] This article is dated December 8, 2008. It is available at the website: http://www.kivitv.com/Global/Story.asp?S=9443255.

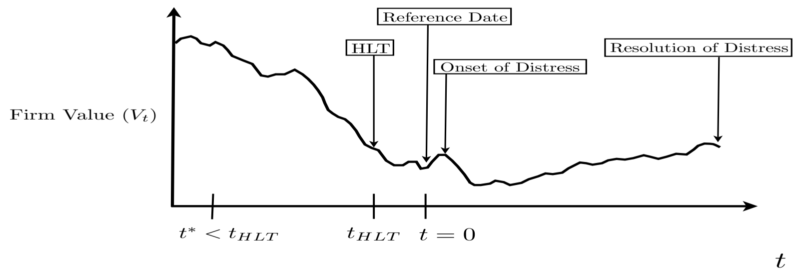

Figure 1: **Timing in Andrade and Kaplan (1998)**. V_t is firm value at time t. The time t_{HLT} refers to the date of a highly levered transaction. "Onset of distress" is the first occurrence of a missed debt payment, attempted restructuring, or when EBITDA was exceeded by interest expense. The reference date, $t = 0$, is the end of the fiscal year immediately preceding the onset of distress.

time. At time t_{HLT}, the firm is involved in a highly leveraged transaction (HLT), which in their study was either a management buyout or leveraged recapitalization. The timing of the HLT is not of any particular significance, other than the fact that it later caused every firm in their sample to enter financial distress, labeled "Onset of Distress." Andrade and Kaplan recognize financial distress as beginning upon the first occurrence of any of the following: the firm missed an interest payment, attempted to restructure its debt, or had EBITDA less than interest expense. Usually, this occurs within 2-3 years of the HLT, but occasionally much sooner (e.g., RJR Nabisco, which restructured its debt only eight months after a management buyout in March 1989).

Their goal is to quantify how much firm value was sacrificed to financial distress, from its onset through its resolution (e.g., bankruptcy proceeds, IPO, etc.). To do this, two values must be calculated. The first is the benchmark value of the firm if it were not distressed. The second is its actual value including distress costs.

To calculate the no-distress benchmark, Andrade and Kaplan multiply the firm's trailing EBITDA by the industry average value-to-EBITDA ratio.[13] Of course, because the timing of distress does not necessarily coincide with the release of accounting information, the authors must use the firm's EBITDA from the end of the fiscal year immediately preceding distress. This is the "Reference Date," or $t = 0$. For example, if a firm whose fiscal year ends in December defaulted in July 1989, the reference date, $t = 0$, would correspond to

[13]This methodology assume that the entire industry is not financially distressed.

December 31, 1988. This convention allows the authors to pin down the value of the no-distress benchmark as close as possible to the onset of distress, subject to having to use accounting information to conduct the valuation.

The second step is to compare the firm's actual $t = 0$ value with this benchmark value. Andrade and Kaplan discount the total capital realized from $t = 0$ through the resolution of distress by the firm's imputed cost of capital (also inferred from industry information).[14] This calculation returns the firm's actual value as of $t = 0$, including the effects of any financial distress costs realized over this period. Thus, if the firm lost sales due to financial distress, this would have reduced its cash flows from $t = 0$ onward, resulting in a lower discounted $t = 0$ value. Andrade and Kaplan (1998) report financial distress costs as a percentage of the benchmark value. For example, if a firm's non-distressed benchmark value were \$100, and its value with financial distress were \$80, the proportional cost of financial distress would be 20%.

However, such an ex-post calculation of financial distress would not, by itself, be useful for a firm standing at some previous point, such as time t^* in the figure. Its value at this time, V_{t^*}, is likely to be substantially different than its value at the onset of distress V_0 due to economic shocks between $t = t^*$ and $t = 0$. Andrade and Kaplan account for this ex-post by benchmarking financial distress losses against V_0, which has already netted out such economic shocks. In the remainder of the paper, we demonstrate how a firm, standing at t^* (or any other arbitrary time), can do this filtering ex-ante. By quantifying the distress threshold V_0 *before* distress happens, the present value of ex-ante financial distress costs can be properly quantified.

2.1 Defining ex-ante financial distress costs

Our formulation for the time t dollar value of ex-ante financial distress costs, FDC_t, is

$$FDC_t = \sum_{j=t+1}^{t+N} \frac{Q_j \cdot LGD_j}{(1+r_j)^j}, \tag{1}$$

where Q_j is the is risk-adjusted probability of defaulting in year j, LGD_j is the dollar loss given default in year j, and r is the risk-free yield to maturity from year t to year j.[15] We forecast FDC for $N = 30$

[14]Because most of Andrade and Kaplan's sample had been taken private by the time they become distressed, their equity values could not be observed directly from stock prices.

[15]LT is set in continuous time, and as such, generates a continuous-time formulation for FDC that is conceptually similar to equation (1), although of infinite horizon, with no significant quantitative differences. We employ a discrete time analog to

years, noting that longer horizons make only a trivial difference. Here we discuss how we estimate each of the required inputs for calculating FDC.

- Q. Following previous studies of ex-ante financial distress, we assume that distress and default states are the same. We infer a 30-year sequence of risk-adjusted default probabilities, $Q_1, Q_2, ..., Q_{30}$, directly from the Leland and Toft (1996) structural model of default. This is an alternative to inferring them by decomposing credit spreads, as in Almeida and Philippon (2007). As we will see, these methods give very similar estimates of risk-adjusted default probabilities. We will provide a more detailed treatment of the model we use later. In the meantime we will, for ease of exposition, denote the firm's asset value V and the value at which it defaults, i.e., the default boundary, V_B.[16] Because the LT model implies a level of V_B in advance as a function of observable parameters, then given dynamics for V, calculating the risk-adjusted probability of hitting the threshold for any future time is straightforward.

- LGD. We obtain the dollar losses in default by multiplying the proportional losses given default by the default threshold, or

$$LGD = \alpha V_B, \tag{2}$$

  where V_B is the default threshold and α is the proportional loss in default. The default threshold V_B arises endogenously in the LT model.[17] Consistent with Davydenko's (2007) empirical estimates, this default boundary corresponds roughly to two-thirds of a firm's total liabilities. In most of our analysis, we take proportional ex-post distress losses, α, directly from Andrade and Kaplan (1998). The range in their paper was 10%-23%. We consider both extremes, but following previous studies, use the midpoint of 16.5% for our benchmark analysis. Later in section 5.2, we discuss how firm-specific LGD can be inferred from market prices.

- r. To construct a risk free term structure, we rely on constant maturity Treasury (CMT) index data from the Federal Reserve. These indices track the most recently issued bonds for each maturity segment.

An alternative formulation for equation (1) is to scale it by current firm value, allowing us to answer the question, "What percentage of ex-ante value is lost financial distress?" This requires nothing more than

preserve expositional simplicity, as well as to compare our results to other studies that have also formulated the problem in discrete time.

[16] In general, the default threshold may be state-dependent. In LT, it is time-invariant.

[17] In alternative specifications such as Merton (1974) or Black and Cox (1976), the firm defaults when it has negative net worth, i.e., V_B equals liabilities. Our results are similar with such alternative default rules.

scaling each side of equation (1) by the firm's current value, V_t:

$$\frac{FDC_t}{V_t} = \sum_{j=t+1}^{t+N} \frac{Q_j \cdot \alpha}{(1+r_j)^j} \cdot \frac{V_{Bj}}{V_t}. \tag{3}$$

Except for the $\frac{V_B}{V}$ term, this formula captures the approaches used in many studies including Molina (2005) and Almeida and Philippon (2007), although the former uses historical instead of risk-adjusted probabilities. We present our results as a percentage of current firm values to allow our results to be compared with these earlier studies.

2.2 Determining the default boundary ex-ante with a structural model

Both a sequence of risk-adjusted probabilities (Q) and default boundaries (V_B) are required to price ex-ante distress costs, but credit ratings give only the former. This is an important distinction because if financial distress is assumed to occur at default, any difference between current firm value and value at default is due to economic shocks, not financial distress. Moreover, the importance of such economic shocks varies significantly across firms, depending on leverage, volatility, and other firm characteristics. To be more specific about these issues requires taking a stand on the firm's value at default. A simple structural model emerges as a natural choice.

However, in addition to quantifying the firm's value at default, a structural approach offers other additional advantages. Compared to using credit ratings to extract probabilities, a structural approach generates firm-specific rather than rating-clustered estimates.[18] This disaggregation allows us to improve precision within ratings categories, but more importantly, to capture differences in systematic risk that lead to different pricing implications between firms with similar default frequencies. Second, our implementation of the structural model naturally allows for a time-varying risk premium, so that a given expected distress loss can have different prices over time. As pointed out by Almeida and Philippon (2007), ignoring time variation in the risk premium when it exists will underestimate the present value of distress costs. We return to this issue in Section 4.

[18] Almeida and Philippon (2007) rely on rating based probabilities whereas others like Berndt et al (2008) and Saita(2007) used Expected Default frequencies (EDFs) provided by Moody's KMV as their estimate of the historical default probabilities. Although EDF data is available at the individual firm level, our study requires a *full term structure* of risk-adjusted and objective probabilities.

2.2.1 Overview of the Leland and Toft (1996) model

We estimate values at default by estimating the Leland and Toft (1996) model.[19] Here, we describe only the timing and structure salient to our analysis, referring the interested reader to the appendix for technical details. In LT, a firm has V dollars in unlevered assets and P dollars in book liabilities. These imply three distinct sources of value: that of the unlevered assets alone, tax benefits from leverage, and financial distress costs should the firm default. Like Merton (1974), Black and Cox (1976), Brennan and Schwartz (1978,) and other papers on structural credit risk models the firm's unlevered assets can be described using a continuous diffusion model with constant proportional volatility, σ. The volatility in the firm's unlevered assets is independent of the firm's financial structure. In other words, the firm's value can increase or decrease due to economic shocks, which is captured by σ.

Financial distress costs are incurred by the firm only upon defaulting, and are proportional to the firm's value at default (see equation 2). An important feature of the LT model is that default is endogenously triggered by shareholders when these no longer find it profitable to prop up the firm by providing additional equity financing. In other words, the default threshold, V_B is endogenous, and will end up depending on most of the parameters in the model - in particular the risk of the firms operations (σ), debt (P) and the ex-post distress costs (α). The appendix provides the technical details. Here, we provide the basic intuition behind how V_B varies with firm characteristics, and how ex-ante financial distress costs FDC are affected.

- Operating risk (σ). When the firm's assets have more volatile values, the effect on V_B is clear. Equityholders own an option on these assets, and will thus be more patient should the firm become economically distressed. Following bad times, equityholders are more likely to fund debt service via equity issuance, delaying the onset of default. This unambiguously pushes down the value of V_B, so that $\left(\frac{\partial V_B}{\partial \sigma} \leq 0\right)$. The effect on ex-ante financial distress costs is less obvious. All else equal, a decrease in V_B makes distress less likely (as firm value is farther from default). However, all else is not equal, as the increase in risk makes hitting a given boundary more likely. The net effect of an increase in operating risk on FDC remains positive, i.e., $\left(\frac{\partial FDC}{\partial \sigma} \geq 0\right)$.
- Debt (P). An increased book value of debt also unambiguously increases default probabilities. As with

[19] A natural first choice for a structural model is the canonical Merton (1974) framework. However, a significant drawback of this model is that it is designed with a simplistic capital structure in mind, with a single debt issue and only one date at which distress can occur - the maturity date of debt. In addition, this model implies a negative net worth covenant as the trigger for financial distress, which as shown in Davydenko (2007) is typically a much higher default boundary than observed in practice. We will return to the Merton model later in our robustness checks but prefer in our main analysis to opt for a model which allows for multiple debt issues, default and financial distress at any point in time and a richer setting for financial distress.

risk, there is a second effect, via the default boundary. However, in contrast to a risk increase, a higher leverage reduces shareholders' patience and thus increases the default boundary. Thus the effect of leverage is amplified by the endogeneity of the default decision. It increases default probabilities and the ex-post dollar loss, thus increasing overall FDC, $\left(\frac{\partial FDC}{\partial P} \geq 0\right)$.

- Proportional loss in default (α). An increase in the ex-post default cost, α, clearly increases the realized distress costs conditional on a given default boundary. However, this realized loss rate also impacts shareholders patience with keeping the firm afloat. A higher loss rate, α, increases the cost of debt financing and thus reduces shareholders willingness to inject funds to avoid distress. This increases the optimal default boundary and thus magnifies the direct effect for a given boundary. In short, a doubling of the loss rate will more than double the ex-ante FDC, $\left(\frac{\partial FDC}{\partial \alpha} \geq 0\right)$.
- Other parameters. For example, increasing the cash payout rate reduces the burden on shareholders to service debt and increases the scope for dividends. This will increase shareholders' patience and slightly reduce the boundary. Yet an increase in the payout comes at the expense of future growth and thus greater future default rates. The net effect is an increase in distress costs, $\left(\frac{\partial FDC}{\partial \delta} \geq 0\right)$. Shareholders' patience increases in the tax rate and the associated subsidy and thus decreases FDC, $\left(\frac{\partial FDC}{\partial \tau} \leq 0\right)$.

In summary it can be said, that in contrast with many quantitative applications in corporate finance and the credit risk literature, the LT model jointly deals with the severity and likelihood of default. Default probabilities *and* the payoffs at default are determined endogenously as a function of shareholders incentives and the firm's environment.

2.2.2 Numerical Example

To earn an appreciation of the economic significance of some of these effects, consider the example in Table 1, which shows three hypothetical firms that pairwise differ in (i) their book liabilities (P) and (ii) their business risk (σ). The quantities presented in these examples have been computed using the LT model. The points we wish to illustrate with these scenarios are how ex-ante and ex-post distress costs for a given proportional loss at default (benchmarked to the upper bound reported by Andrade and Kaplan (1998)) depend on leverage and volatility.

Consider first the impact of a change in leverage. Firm A has book liabilities of \$35 and firm B \$55. Clearly the shareholders of a firm with a higher burden in terms of debt service will yield control of the firm to

	Firm A	Firm B	Firm C
Asset volatility	30.00%	30.00%	20.00%
Book value total liabilities (P)	35.00	55.00	35.00
Interest on debt (C)	1.75	1.75	1.75
Expected firm growth rate	15.00%	15.00%	15.00%
Objective ten year default probability	3.42%	11.84%	0.06%
Risk-adjusted ten year default probability	16.51%	34.75%	3.32%
Market value tax shield ($)	3.71	4.76	4.63
Endogenous default threshold	22.05	34.66	26.01
Ex post distress costs ($)	5.07	7.97	5.98
Ex post distress costs as % of value at default	23.00%	23.00%	23.00%
Ex ante distress costs ($)	1.49	3.37	0.71
Ex ante distress costs as % of firm value	1.45%	3.32%	0.68%

Table 1: **The role of leverage and asset volatility in determining the ex-ante and ex-post costs of financial distress**. Across the three firms we set the initial asset value to 100, the risk free rate to $r = 5\%$, the ex-post proportional loss to $\alpha = 0.23$ motivated by the upper bound reported by Andrade and Kaplan (1998), the payout rate ($\delta = 3\%$) motivated by the average total payout in our sample, the maturity of newly issued debt to $T = 10$ and the effective corporate tax rate to $\tau = 15\%$

creditors at a higher asset value. This is reflected in an increase in the default threshold V_B from $22 to $35. This increases the ex-post distress cost from about $5 to just less than $8. The risk-adjusted default probability increases as a direct result of this from 16.5% over a 10 year horizon to almost 35%. Firm A corresponds loosely to a firm with a rating in the lower A to higher BBB categories when compared on a risk-adjusted default probability basis to Table 3 in Almeida and Philippon (2007).[20] Firm B is similar to a BB rated firm in their table.

This translates into ex-ante costs of 1.45% and 3.32% respectively. In Almeida & Philippon (2007) the numbers would be significantly higher at approximately 5.5% and 9% respectively. The reason for this discrepancy lies in the differences in the assumed firm value at default, which when multiplied by Andrade and Kaplan (1998) upper estimate of 23%, discounted and deflated by the risk-adjusted default rate yields the ex-ante cost of distress. They consider implicitly firms that are near default whereas our firms will only default after significant deteriorations in asset value (from 100 to 22 and 35 respectively).[21]

A further important point is that even for a given amount of book liabilities, the operational risk of the firm will be a first order determinant of the ex-ante costs of distress. As mentioned above, there are two effects. First, for a given level of required debt service, higher risk raises the risk of distress. The second effect is that higher (lower) risk will increase (decrease) the option value of the shareholders' equity stake. Decreased

[20]The objective 10 year default rate implied by the LT model lies at 3.42% which lies in the same rating band as our risk-adjusted numbers when compared to Table 3 of Almeida and Philippon (2007).

[21]In fact, the ratio between our estimates of ex-ante costs and theirs is approximately equal to the ratio of current asset value to value at default (V_B) in our example.

risk will thus influence shareholders to default at a higher firm value. In our example, Firms A and C have the same book liabilities, but differ in their asset volatility. Firm C with a lower risk of 20%, has a higher default threshold (V_B) of \$26 compared to firm A's \$22. This entails, for a fixed loss rate of 23%, a higher ex-post cost of distress by about 18%. Nonetheless, the overall effect is to decrease the default probability and thus the ex-ante costs of distress from 1.45% to 0.68% of initial levered firm value.

Overall these simple examples highlight the importance of accounting for the level of firm value at which the firm is expected to default. In addition, it becomes clear that the probability of default cannot be treated in isolation from what happens at the default date. The LT model allows a straightforward measurement of these indirect effects in both ex-post and ex-ante terms.

2.2.3 Estimation

In order to implement this framework more generally in the cross-section, we exploit that the Leland and Toft (1996) model admits closed form solutions both for the value and the volatility of a firm's equity. For a given set of firm parameters describing debt structure and other firm characteristics, we can solve for the value and volatility of the firm's unlevered assets at each quarter in our data, by requiring that the model matches the firm's observed market capitalization and an estimate of equity volatility. These estimates of value and volatility (V_t, σ) can then be used to compute term structures of historical and risk-adjusted default probabilites, default boundaries and consequently also levels of ex-ante costs of financial distress (FDC).

Most of the inputs such as leverage, firm values, and payout rates are obtained from the quarterly COMPUSTAT tapes for every firm from 1971 - 2007 with sufficient data to conduct the estimation. Our sample moments for a variety of characteristics including market leverage, book leverage, profitability, tangibility, and cash flow volatility are nearly identical to those reported in Lemmon, Roberts, and Zender's (2008) comprehensive study of capital structure. In addition to these, we construct a rolling 250-trading day equity volatility for each firm-quarter observation in our data set from the CRSP files, which we then use as an input to the model. For tax rates, we employ two approaches. First, for about 85% of the firms in our sample, we have obtained average forward-looking marginal effective tax rates from John Graham's website. Averaged across all firm-quarters, the average tax rate is 19.96%. For some of our analysis, we use these. For others, we use a constant tax rate of 20%, ignoring cross-sectional variation between firms. Either approach gives very similar estimates.

We describe the methodology in more detail in the appendix, but overall the implementation methodology is quite standard. It has been used since Jones, Mason, and Rosenfeld (1984) and was recently used in Bharath and Shumway (2008). Furthermore, in advance of our results, we note that the LT model implemented with the above assumptions generates historical default rates strikingly similar to those used in Almeida and Philippon (2007). We now discuss these and our main results.

3 Results

We divide our results into two main categories. In subsection 3.1, we quantify the overall levels of ex-ante distress costs and contrast them with previous studies. Then, in subsection 3.2, we examine the implications for target capital structure. Specifically, we compare the leverage sensitivities of ex-ante financial distress costs versus the leverage sensitivities of tax benefits. When the marginal benefit of debt exactly offsets the marginal cost, these sensitivities should equate.

3.1 Levels

For most firms, their current values (V_t) are quite far from their values at default (V_B), when financial distress would occur. Thus, $\frac{V_t - V_B}{V_t}$ is the percentage economic shock that a firm at time t would have to sustain before suffering distress costs. Table 2 presents current firm values for every firm-quarter observation in our sample, default thresholds predicted by the LT model, as well as each firm's total liabilities (*Debt*) and asset volatility (σ).

The values reported in the table are medians (unless otherwise indicated) for quintiles defined on Moody's-KMV distance-to-default (henceforth DTD).[22] The advantage of this metric is that it allows us to stratify simultaneously along financial leverage and unlevered volatility, even for firms without public debt ratings. Firms in the lowest quintile have an average DTD of 2.4, which can be interpreted as being 2.4 asset value standard deviations away from financial distress over a one-year horizon. These firms have 5-year objective cumulative default rates of about 30% which corresponds roughly to rates for single B rated firms by Moodys. By the same token, our highest DTD quintile corresponds roughly to the AA/AAA category.

[22] $DTD = \frac{ln(V_t) - ln(V_B)}{\sigma}$, where V_t is the firm's current asset value, V_B is the default threshold, and σ is the asset volatility.

DTD	2.4	3.6	4.6	5.8	8.9	All (median)	All (mean)
V_t $(MM\$)$	35.6	82.2	155.7	270.3	454.2	135.2	1,917.4
V^L $(MM\$)$	37.0	86.5	163.7	281.7	468.8	141.5	2.027.8
$Debt$ $(MM\$)$	21.2	33.1	47.3	57.8	60.0	38.2	727.1
$\frac{Debt}{V_t}$	0.60	0.40	0.30	0.21	0.13	0.28	0.38
σ	0.42	0.36	0.33	0.31	0.25	0.32	0.41
V_B $(MM\$)$	11.0	19.2	28.5	36.6	40.6	23.0	561.6
$\frac{V_B}{V_t}$	0.31	0.23	0.18	0.14	0.09	0.17	0.29
N	90,455	90,455	90,455	90,455	90,455	452,274	452,274

Table 2: **Default thresholds predicted by LT model**. This table shows the default thresholds predicted by the Leland and Toft (1996) model. Unless otherwise specified, all values are medians. DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. The first five column are quintiles of DTD (means) for the entire sample. V_t is asset value and V^L denotes levered firm value. Unlevered asset volatility is σ, and the default threshold is V_B.

The leftmost column refers to the riskiest 20% of our observations, with credit quality increasing as one moves to the right. The first and second row reports current firm market values and total debt, respectively, for every firm-quarter observation in our sample. Both are highly right-skewed. The mean (median) firm value is \$1.9B (\$135M), with \$727.1M (\$38.2M) in total debt. The firms closest to default tend to be smaller, more leveraged (row 3), and more volatile (row 4). All of these patterns are well-known in the credit literature.

We are mainly interested in the quantities shown in the final two rows. The fifth row shows the default threshold, V_B, predicted by the LT model. The last row scales this default threshold by the firm's current value. As seen, the median ratio of V_B to V_t is roughly 0.17, indicating that the median firm could lose over 80% of its value before defaulting. In virtually all cases, the default threshold is less than the firm's debt. This occurs because equity has option value - equity holders have an incentive to inject equity into a struggling firm if there is sufficient upside. However, Table 2 also makes clear that even without such optionality, values at default are quite different from current firm values. For example, suppose the firm is required to default as soon as its value V drops below the face value of its total debt (similar to Merton (1974)). The third row indicates that even in this case, the median firm would have to lose over 70% of its value before defaulting. For the remainder of the paper, we maintain the LT convention that default is triggered at the first time when firm value drops below V_B, but note that alternative default rules will give similar results (see section 5.4).

In addition to an estimate of the default threshold V_B, a sequence of risk-adjusted default probabilities is also required in order to calculate ex-ante distress costs. Table 3 shows both the 5-year and 10-year probabilities

DTD (mean)	2.4	3.6	4.6	5.8	8.9
P_{t+5}	0.322	0.107	0.034	0.007	0.000
P_{t+10}	0.464	0.225	0.110	0.040	0.006
Q_{t+5}	0.399	0.180	0.077	0.021	0.001
Q_{t+10}	0.573	0.372	0.240	0.123	0.025
Moodys Rating	B	BB	BBB	A	AA/AAA
P_{t+5}^{Moodys}	0.310	0.114	0.020	0.005	0.002
P_{t+10}^{Moodys}	0.465	0.215	0.052	0.016	0.009
Q_{t+5}^{AP}	0.349	0.211	0.114	0.071	0.011
Q_{t+10}^{AP}	0.625	0.392	0.209	0.127	0.042

Table 3: **Benchmarking LT default probabilities.** This table compares the objective and risk-adjusted default probabilities predicted by the Leland and Toft (1996) model to Moody's historical averages and to Almeida and Philippon's (2007) estimates, respectively. Unless otherwise specified, all values are medians. DTD is KMV-Moody's distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. The columns are broken up by quintiles of DTD (means) for the entire sample. The first two rows present five and ten-year objective default probabilities predicted by LT, P_{t+5} and P_{t+10} respectively. The third and fourth rows present their risk-adjusted counterparts, Q_{t+5} and Q_{t+10} respectively. For comparison, rows five and six show the objective default probabilities from Moody's over 1971-2001 (to be compared to rows one and two). Rows seven and eight are Almeida and Philippon's (2007) risk-adjusted probabilities of default. Rows five through eight are presented by Moody's credit rating.

of default predicted by the LT model, broken down by credit quality as in Table 2. The first and third rows of Table 3 are objective (historical) default probabilities, while the second and third rows are those adjusted for systematic risk. For comparison, directly underneath we show Moody's estimates (1970-2001) of objective default probabilities as well as Almeida and Philippon's (2007) estimates of risk-adjusted default probabilities, each by Moody's credit rating: B, BB, BBB, A, AA/AAA.

Two things are apparent in Table 3. First, the objective probabilities generated by the LT model are quite reasonable, and when stratified by DTD quintiles, line up closely with Moody's objective default probabilities. Although these objective probabilities do not enter into either pricing equation (1) or (3), we show them to emphasize that the model realistically captures the important determinants of default. Ultimately, we will use outputs from the model that we *cannot* benchmark against historical averages, as we can with objective default probabilities. The fact that the model performs well in this dimension is reassuring.[23]

The second noteworthy observation is that the risk-adjusted default probabilities generated by the LT model

[23]See Leland (2004) for a more detailed discussion regarding the ability of the LT model to match objective default likelihoods.

DTD	2.4	3.6	4.6	5.8	8.9	All (median)	All (mean)
$\frac{FDC}{V^L}$	0.020	0.009	0.005	0.002	0.000	0.004	0.009
$\frac{FDC}{V^L}$ with $V_B = V_t$	0.086	0.054	0.038	0.025	0.006	0.035	0.045
Δ	0.066	0.045	0.036	0.023	0.006	0.031	0.036
V_t $(MM\$)$	35.6	82.2	155.7	270.3	454.2	135.2	1,917.4
V_B $(MM\$)$	11.0	19.2	28.5	36.6	40.6	23.0	561.6
N	90,455	90,455	90,455	90,455	90,455	452,274	452,274

Table 4: **Ex-ante financial distress costs by distant to default,** $\alpha = 0.165$**.** This table shows the present value of ex-ante financial distress costs implied by the LT model. Unless otherwise specified, all values are medians. DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. The first five column are quintiles of DTD (means) for the entire sample. The first row, $\frac{FDC}{V^L}$, is the present value of ex-ante distress costs capitalized and discounted, as a fraction of current firm value. The second row presents the results of estimating equation (3), but does not filter out the effects of future economic shocks. Row three is the difference between the first two rows. V^L is total firm value, V_t current asset value and the default threshold is V_B.

are strikingly similar to those Almeida and Philippon (2007) extract from credit spreads. This is particularly surprising given the substantial differences in methodology and sample firms (i.e., publicly rated vs. all CRSP/COMPUSTAT firms). As we discuss in section 4, the LT model explicitly accounts for time-varying risk premia in the credit markets, which Almeida and Philippon (2007) admit is difficult to incorporate into their methodology. However, the similarities between their risk-adjusted probabilities and ours make quite clear that their estimates (at least on average) do not suffer bias from the inability to account for a time-varying risk premium.

Our main results are shown in Table 4 which, as before, are presented separately by default risk. All estimates are based on the midpoint of Andrade and Kaplan's (1998) study of 16.5% proportional losses ex-post to default. The first line contains the median ex-ante financial distress costs, scaled as a percentage of current firm value (equation 3). As seen, the median firm in our sample sacrifices approximately one-half percent of its value in expected financial distress costs. The mean is below 1% of firm value. Only among firms closest to default (leftmost column) do ex-ante distress costs exceed 2% of pre-distress value.

In the second row, we show the ex-ante distress costs *without* filtering out economic shocks between current firm value and value at default, i.e., assuming $V_t = V_B$. In every case, this produces much higher estimates. In the riskiest group, ex-ante financial distress costs are in the neighborhood of 8%-9%, remaining substantial for all but the safest quintile across row 2. Unsurprisingly, row 2 closely resembles Almedia and Philippon's estimates of ex-ante financial distress costs, which by rating category are 9.54% (B), 6.81% (BB), 4.53%

(BBB), 3.83% (A), 1.84% (AA), and 0.32% (AAA).[24] Given that ours and their risk-adjusted default probabilities are so close, and because we both use Andrade and Kaplan's midpoint of 16.5% for proportional ex-post losses, we would expect similarity in ex-ante distress costs.

The final two rows shows why the first and second rows - or alternatively, why our first row and Almeida and Philippon's estimates - differ so markedly. Losses between the firm's current value and default threshold are due to *economic* shocks, not financial distress. As such, the dollar losses $V_t - V_B$ should not be included in the calculation of ex-ante financial distress costs. Going across Table 4, calculations of financial distress costs that ignore this are increasing biased. The riskiest firms are expected to default after losing two-thirds of their current values, so that the calculations in row 2 are too high by a factor of over four. For the safest firms that are far from defaulting, ex-ante financial distress costs are over an order of magnitude too high. It is worth re-emphasizing that the differences between the first and second rows (or alternatively, between the first row and Almeida and Philippon's estimates) are not due to differences in risk-adjusted default probabilities. As indicated in Table 3, these are nearly identical. Instead, it is distinguishing between economic shocks and financial distress that leads to the stark differences.

Tables 5 and 6 show the results for higher and lower estimates for proportional ex-post recovery rates, $\alpha = 0.10$ and $\alpha = 0.23$, respectively. As expected, compared to those in Table 4, the estimates in Table 5 (Table 6) are lower (higher). Although we do not take a stand on an appropriate level for α, it is clear that only for the upper end of Andrade and Kaplan's (1998) range does financial distress impose a meaningful impairment to firm value, and even then, only for the riskiest firms. Because the firms in this quintile are very small, the market value of the deadweight costs of financial distress is almost negligible.

Perhaps unsurprisingly, the ratios of $\frac{FDC}{V_t}$ between any pair of tables in 4, 5, or 6 is very close to the ratio of their respective value of α. For example, the ratios of $\frac{FDC}{V_t}$ between Table 4 and Table 6 are, for each risk class respectively, 1.396, 1.433, 1.444, 1.453, and 1.475, while the ratio of α is $\frac{.23}{.165} = 1.393$. But why are there any discrepancies at all? And why do these discrepancies increase as one moves from the left to right, i.e., for increasing levels of credit quality?

The answer is that in the LT model, changes in proportional ex-post losses, α, has multiple effects. Revisiting equations (1) and (2), and taking the derivative with respect to α, we have:

[24] See Table IV, Panel A of Almeida and Philippon (2007), p. 2571.

DTD	2.5	3.6	4.6	5.9	9.0	All (median)	All (mean)
$\frac{FDC}{V^L}$	0.012	0.005	0.003	0.001	0.000	0.003	0.005
$\frac{FDC}{V^L}$ with $V_B = V_t$	0.052	0.032	0.023	0.014	0.003	0.021	0.027
Δ	0.040	0.027	0.020	0.013	0.003	0.018	0.022
V_t $(MM\$)$	35.0	82.1	156.0	271.4	461.3	135.1	1,915.8
V_B $(MM\$)$	10.3	18.6	27.8	36.2	41.0	22.4	548.0
N	90,455	90,455	90,455	90,455	90,455	452,274	452,274

Table 5: **Ex-ante financial distress costs by distant to default,** $\alpha = 0.10$**.** This table shows the present value of ex-ante financial distress costs implied by the LT model. Unless otherwise specified, all values are medians. DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. The first five column are quintiles of DTD (means) for the entire sample. The first row, $\frac{FDC}{V^L}$, is the present value of ex-ante distress costs capitalized and discounted, as a fraction of current firm value. The second row presents the results of estimating equation (3), but does not filter out the effects of future economic shocks. Row three is the difference between the first two rows. V^L is total firm value, V_t unlevered asset value and the default threshold is denoted V_B.

$$dFDC = \underbrace{\frac{\partial FDC}{\partial \alpha} d\alpha}_{\text{direct effect}} + \overbrace{\frac{dFDC}{dV_B} \cdot \frac{dV_B}{d\alpha} d\alpha + \sum_{j=t+1}^{t+N} \frac{\partial FDC}{\partial Q_j} \cdot \frac{dQ_j}{d\alpha} d\alpha}^{\text{indirect effect}} \tag{4}$$

The first is the obvious direct and positive effect. Holding all else equal, increasing the fraction lost in distress increases the firm's ex-ante costs in direct proportion. The second is related to the optionality of equity and is embedded in structural models like LT, in which default is an endogenous decision made by shareholders. With increased proportional losses in default (increases in α), debtholders expect to recover a smaller fraction of their investment ex-post, which causes them to charge a higher rate ex-ante. This higher interest expense in turn increases the cost of keeping the firm alive, or alternatively, reduces the present value of the call option the equityholders hold on the firm's assets. Being less patient, equityholders default more quickly, which increases the default threshold, V_B.

As equation (4) makes clear, a higher default threshold has two effects on the ex-ante distress costs. The first term under the heading "indirect effect" indicates that a higher default threshold means higher dollar losses in distress, e.g., losing 20% of \$100M is more costly than losing 20% of \$80M. The second indirect effect is that default is more likely for a higher default threshold, e.g., default is more likely if firm value need only drop to \$100M rather than \$80M. Because both effects increase α, the indirect effect is always positive. Thus, doubling a firm's proportional losses in default will *more* than double its ex-ante financial

DTD	2.4	3.5	4.5	5.8	8.8	All (median)	All (mean)
$\frac{FDC}{V^L}$	0.029	0.013	0.007	0.003	0.001	0.006	0.013
$\frac{FDC}{V^L}$ with $V_B = V_t$	0.120	0.075	0.054	0.035	0.009	0.049	0.063
Δ	0.091	0.062	0.047	0.032	0.008	0.043	0.052
V_t $(MM\$)$	36.1	82.5	155.5	268.9	446.2	135.3	1,920.1
V_B $(MM\$)$	11.6	19.9	29.1	37.0	40.1	23.5	575.9
N	90,455	90,455	90,455	90,455	90,455	452,274	452,274

Table 6: **Ex-ante financial distress costs by distant to default,** $\alpha = 0.23$**.** This table shows the present value of ex-ante financial distress costs implied by the LT model. Unless otherwise specified, all values are medians. DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. The first five column are quintiles of DTD (means) for the entire sample. The first row, $\frac{FDC}{V_t}$, is the present value of ex-ante distress costs capitalized and discounted, as a fraction of current firm value. The second row presents the results of estimating equation (3), but does not filter out the effects of future economic shocks. Row three is the difference between the first two rows. V^L is total firm value, V_t is unlevered asset value and the default threshold is V_B.

distress costs.

The strength of this effect depends on other model parameters. For example it is stronger for lower risk firms. The intuition for this result is the following. At high levels of volatility, the option is very valuable. Increases in α still make shareholders less patient, but because they are reluctant to give up the option value, V_B - the point where they are willing to throw away the option - is difficult to budge. The converse is true for low levels of volatility, where relatively small changes in the cost of debt (through α) can have large impacts on V_B. Consequently, the indirect effect in 4 is larger for stable firms with low volatility. Figure 2 plots the model implied levels of $\frac{dFDC}{dV_B} \cdot \frac{dV_B}{d\alpha}$ against the level of the loss given default parameter α for different levels of the asset volatility σ. The asset risk of a firm clearly diminishes the impact of this indirect effect.

While here we have found that levels of ex-ante distress costs appear relatively small, this is only suggestive evidence of underleverage. Ultimately, firms care about how these costs vary with leverage.[25] Graham (2000) shows that most firms could significantly increase the present value of their tax shield by increasing their debt usage. This is optimal only if ex-ante financial distress costs do not increase at a magnitude sufficient to offset this gain. We turn to this issue next.

[25]Evidence reported by Graham & Harvey (2001) suggests that managers place significant emphasis on credit ratings - and by implication the likelihood of financial distress - when making decisions about their financial and operating policies. From this perspective, it is also interesting to study the impact of changes in volatility and leverage on default rates.

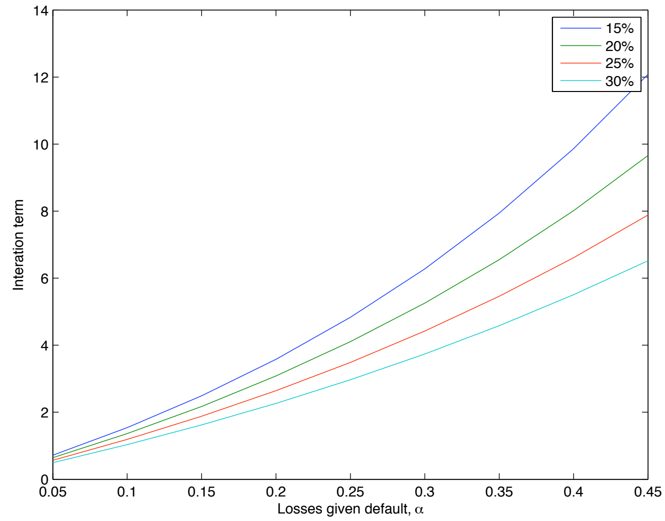

Figure 2: **The indirect effect of a shock to losses given default.** This figure plots the absolute size of the interaction term $\frac{dFDC}{dV_B} \cdot \frac{dV_B}{d\alpha}$ in equation (4) as a function of the level of losses given default (α) for various levels of asset volatility (σ). We set the initial asset value to 100, the risk free rate to 5%, the average debt maturity to $T = \frac{10}{2}$, the cash payout rate to 6%, the corporate tax rate to 15%, the aggregate debt service to $C = 5$, $P = 50$.

3.2 Sensitivities

We have so far compared the levels of FDC to estimates in the literature, in particular to those recently documented in Almeida and Philippon (2007). Here we ask, "How do our estimates of the marginal effects of leverage on distress costs compare?" To address this, consider an example from Table IV in Almeida and Philippon (2007). They find that a firm with A rating and corresponding leverage ratio of 22% has FDC of 3.83%. A BB firm, with a 34% leverage ratio has FDC of 6.81%. This is an increase of about 0.25% per percentage point increase in leverage.

Now consider our Table 4. Although we do not have comprehensive rating data, we have shown (in Table 3) that our second and fourth distance-to-default quintiles correspond approximately to A and BB rating categories when comparing our historical and risk-adjusted default probabilities. Our typical A firm value suffers 0.2% in FDC, whereas a BB firm's value is reduced by 0.9%. This entails an increase in FDC of 0.7% for an increase in leverage by 9% points (see Table 2), or a 0.08% per leverage percentage point increase. Even this rough estimate of our sensitivity is about a third of the corresponding metric in Almeida and Philippon (2007). Like the level results, this reduction in sensitivity is due to the fact that our estimates are purged from the effects of economic shocks.

DTD (mean)	2.4	3.6	4.6	5.8	8.9	All (mean)	All (median)
Q_{t+5}	0.399	0.180	0.077	0.021	0.001	0.136	0.046
$\frac{\partial FDC}{\partial P}$	0.082	0.059	0.045	0.034	0.018	0.048	0.039
$\frac{\partial TB}{\partial P}$	0.094	0.125	0.141	0.156	0.178	0.139	0.148
Δ	0.011	0.066	0.096	0.122	0.159	0.091	0.109
N	90,455	90,455	90,455	90,455	90,455	452,274	452,274

Table 7: **Sensitivities of distress costs, tax benefits and levered firm value to increases in book debt,** $\alpha = 0.165$. This table shows the sensitivities of ex-ante financial distress costs to changes in debt, as well as the sensitivities of tax benefits to leverage. Unless otherwise specified, all values are medians. DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. The first five column are quintiles of DTD (means) for the entire sample. Q_{t+5} is the risk-adjusted default probability predicted by the LT model. $\frac{\partial FDC}{\partial P}$ is the slope of financial distress costs with respect to debt (P), and $\frac{\partial TB}{\partial P}$ is the slope of tax benefits with respect to debt. All sensitivities are dollar-for-dollar sensitivities.

To provide a more rigorous examination of this issue - but stopping short of a full scale measurement of firm specific tax shield values - we carry out the following exercise. We compute, for the Andrade and Kaplan (1998) endpoints and midpoint for proportional distress losses (10%, 16.5%, and 23%) and a 20% level for the effective marginal corporate tax rate, the numerical derivatives of FDC $\left(\frac{dFDC}{dP}\mid_{V_t,\sigma}\right)$ and tax benefits $\left(\frac{dTB}{dP}\mid_{V_t,\sigma}\right)$ with respect to book debt levels, keeping estimated unlevered asset values and volatilities constant. At the optimal leverage, these two derivatives should equate. Here, we use the sample average of $\tau = 20\%$ for all firms, but model firm-specific tax rates in section 5.3.

The results of this exercise are presented in Table 7. The most risky firms (the 20% with the lowest distance-to-default) would stand to increase their distress costs by 8.2 cents for each additional dollar of debt issued, which is not quite sufficient to offset the increased tax benefits of 9.4 cents. As we consider progressively safer firms, we find that distress costs sensitivities are less and less able to balance those of the tax benefits. For the safest firms a 1 million dollar increase in debt would entail an increased tax shield by 178 thousand dollars, but only increased distress costs of 18 thousand dollars.

Consider our typical firm. In our sample, the leverage ratio of the "median firm" turns out to be 28% (see Table 2). This firm has total liabilities of \$38MM, a value of \$135MM and would need to increase debt by \$1.35MM to increase its leverage by 1%. A decision to do so would, according to our Table 7, increase FDC by about \$52K or about 0.4%, lower yet than the rough estimate above. Our estimates of the sensitivities are significantly lower than those reported in Almeida and Philippon (2007), at between a third or a sixth

DTD (mean)	2.4	3.6	4.6	5.8	8.9
Q_{t+5}	0.399	0.180	0.077	0.021	0.001
$\frac{\partial FDC}{\partial P}$	0.047	0.034	0.026	0.019	0.011
$\frac{\partial TB}{\partial P}$	0.094	0.125	0.141	0.157	0.178
Δ	0.047	0.091	0.115	0.137	0.168

Table 8: **Sensitivities of distress costs, tax benefits and levered firm value to increases in book debt** $\alpha = 0.10$. This table shows the sensitivities of ex-ante financial distress costs to changes in debt, as well as the sensitivities of tax benefits to leverage. Unless otherwise specified, all values are medians. DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. Columns are broken down by quintiles of DTD (means) for the entire sample. Quintiles of DTD (means) for the entire sample. Q_{t+5} is the risk-adjusted default probability predicted by the LT model. $\frac{\partial FDC}{\partial P}$ is the slope of financial distress costs with respect to debt (P), and $\frac{\partial TB}{\partial P}$ is the slope of tax benefits with respect to debt. All sensitivities are dollar-for-dollar sensitivities.

of their figures.

We now turn to the sensitivities of tax shields. Again we will compare these to the benchmark figures reported by Almeida and Philippon (2007). Their A and BB firms have tax benefits worth 4.4% and 7.2% respectively. This implies, given the leverage ratios above, an increase in the tax shield of 0.24% per percentage point increase in leverage. Returning to our "median firm," our estimate of $\frac{\partial TB}{\partial P}$ is 0.148, meaning that for a one percent increase in leverage there would be a about 0.15% increase in the value of the tax shield. This is somewhat lower than the sensitivity reported in Almedia and Philippon (2007), likely due to the fact that our tax benefits are computed using risk-adjusted default probabilities. However, despite the fact that this risk adjustment lowers the values and sensitivities of tax benefits, we see that financial distress costs are still unable to offset them. Using Almeida and Philippon's (2007) tax benefits unadjusted for risk, and the difference in sensitivities becomes even more stark.

Clearly, different assumed losses in default will alter these sensitivites. Tables 8 and 9 repeat the above analysis for the upper and lower bounds on of the Andrade and Kaplan (1998) estimates of 10% and 23% respectively. Considering the lower bound first we find, not surprisingly, more pronounced evidence of debt conservatism. For the safest firms that can increase their tax shield by 178 thousand dollars for each million of debt issued, the increase ex-ante costs of distress are only 11 thousand dollars. For the riskiest quintile, there firms are still significantly underleveraged - incremental benefits of debt are more than twice the marginal costs of distress.

DTD (mean)	2.4	3.6	4.6	5.8	8.9
Q_{t+5}	0.399	0.180	0.077	0.021	0.001
$\frac{\partial FDC}{\partial P}$	0.123	0.085	0.065	0.049	0.027
$\frac{\partial TB}{\partial P}$	0.094	0.125	0.140	0.155	0.177
Δ	-0.028	0.039	0.075	0.106	0.150

Table 9: **Sensitivities of distress costs, tax benefits and levered firm value to increases in book debt** $\alpha = 0.23$. This table shows the sensitivities of ex-ante financial distress costs to changes in debt, as well as the sensitivities of tax benefits to leverage. Unless otherwise specified, all values are medians. DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. Columns are broken down by quintiles of DTD (means) for the entire sample. Q_{t+5} is the risk-adjusted default probability predicted by the LT model. $\frac{\partial FDC}{\partial P}$ is the slope of financial distress costs with respect to debt (P), and $\frac{\partial TB}{\partial P}$ is the slope of tax benefits with respect to debt. All sensitivities are dollar-for-dollar sensitivities.

It is only when we consider the case where ex-post costs of distress are assumed to be 23% of unleveraged firm value in distress that we find any evidence of overshooting. In the case of the riskiest quintile, we do find that increasing leverage would decrease firm value on average. For the remaining quintiles we find as for the other two distress costs levels that the model implied benefit of increasing leverage outweighs the incremental distress costs incurred ex-ante.

So far, we have documented that most firms in our sample appear to use debt more conservatively than the trade-off theory would predict, confirming Graham's (2000) conclusion. We noted above that more than 90% of our firm-quarter observations with $\alpha = 0.165$ correspond to situations where an increase in book debt values would increase firm value more than the ensuing distress costs would lower them. However, we have not been able to say *by how much* firms are in fact under-leveraged.

Although a detailed measurement of target leverage ratios is not our primary goal, we will now conduct this exercise for our median firm. We use median values for the asset value and volatility, payout rates and other parameters across our sample. The LT model permits computing a solution to the quantitative trade-off between the present value of tax benefits and FDC. We consider different levels of the corporate tax rate ranging from 15% (the level used by Almeida and Philippon (2007)) to 35%. Each of these tax scenarios is combined with one of the three ex-post distress cost scenarios studied previously (10%, 16.5% and 23%). Table 10 reports the optimal market leverage ratios.

For the midpoint distress cost of 16.5% the optimal leverage ratios implied by the LT model range from 34% to 51%. The actual leverage ratio of 28% can be reconciled with the model only if we use the lowest tax rate and highest distress costs. For all other combinations the theoretical leverage ratios exceed the actual by between 6% and 30%.

As mentioned, a full characterization of target leverage is beyond our current scope. This would require at the very least a model where capital structure can be rebalanced as the firm experiences shocks to value, volatility and other characteristics. If shareholders retain the option to lever up in the future, then they will select a lower level at the outset, suggesting the numbers above are biased upwards.[26] In addition, the treatment of the tax shield valuation is overly simplistic. Ideally one would, as in Graham (2000) require firm specific state dependent tax rate. However, the present exercise is instructive insofar as it allows for "pure" financial distress costs to be compared to tax benefits, both of which are adjusted for systematic risk. Our evidence so far suggests that when just these factors are considered, firms generally appear to use leverage too conservatively. However, this is not always so. In the next section, we characterize conditions under which financial distress costs can be large compared to the tax benefits.

4 Cross-sectional and time series properties of distress costs

Our analysis so far has shown that for the vast majority of firms, the levels and sensitivities of ex-ante financial distress costs are too small to reconcile observed leverage ratios with the traditional version of the trade-off theory. However, this is not always the case. As Almeida and Philippon (2007) point out, because "financial distress is more likely to occur in bad times (p. 2557)," its ex-ante cost depends on the risk premium. This insight generates two distinct empirical predictions. First, because the credit risk premium exhibits significant time variation, we can identify times when ex-ante distress costs are highest. Second, holding all else equal, firms with high systematic risk will have high ex-ante distress cost.

A significant advantage of Almeida and Philippon's approach is its parsimony, requiring only observable credit spreads and a model for breaking it into components. If one can quantify compensation for liquidity, expected losses, and the risk premium, then obtaining a sequence of risk-adjusted probabilities from credit spreads is straightforward. Almeida and Philippon (2007) illustrates this methodology. However, the authors also note that although elegant, their approach does not easily accommodate potentially important features of credit spreads that are relevant for the calculation of ex-ante distress costs. Perhaps the largest concern is not fluctuation in the credit spread itself, but in the *proportion* due to the risk premium. As documented

[26] See, e.g., Chen (2008), Fischer, Heinkel, and Zechner (1989), Strebulaev (2007) or Tsyplakov and Titman (2007).

	$\alpha = 10\%$	$\alpha = 16.5\%$	$\alpha = 23\%$
Tax rate 15%	44%	34%	26%
Tax rate 25%	53%	44%	37%
Tax rate 35%	58%	51%	44%

Table 10: **Model-implied optimal leverage ratios.** As inputs we have used the median values for firms in our sample. The median asset value $V_t = 135$ (\$MM), the asset volatility $\sigma = 0.321$, the book debt value $P = 38$, the payout rate $\delta = 0.029$, the maturity of newly issued debt is $T = 6.76$, while the tax rates and proportional ex-post distress costs are varied. The actual market leverage ratio for this "median firm" is 28%.

in Berndt et al. (2008) and Elkhami and Ericsson (2008), the risk premium in credit markets is highly time-varying. Consequently, credit spreads can be over 70% attributable to expected losses, with relatively limited compensation for risk (e.g., 2000); conversely, when the risk premium is high, more than three quarters of the credit spread can be due to compensation for bearing systematic risk (e.g., early 1990s). Almedia and Philippon (2007) demonstrate that ignoring time-variation in the risk premium will understate the risk adjustment to the objective probabilities, leading to estimates of financial distress costs that are too low.

As we study a firm-level panel over more than thirty years, we are in a position to document a number of stylized facts regarding historical and risk-adjusted default rates and ex-ante distress costs. In doing so, we are able to ask to what degree time variation in risk premia matters for the time variation in distress costs. Consider the the 5-year default rates summarized in Figure 3. The most striking impression is the degree of time variation in these - note the peaks in default probabilities after the 1973 oil shock, the 1987 stock market crash and in 2000 and 2001, a period of unprecedented defaults. The wedge between the risk-adjusted and historical default probabilities increases persistently from 1980 to mid 2000. There are peaks again in 1987 and 2000. This is when the risk-adjustment emphasized in Almeida and Philippon (2007) will matter the most.

Figure 4 translates our time varying term structures of default probabilities into a time series of FDC with and without risk-adjustment. We note peaks for the FDC in mid 1974, late 1987 and late 2002. Almeida and Philippon (2007) observe that time variation in bond spread implied default rates - historical and risk-adjusted - may yield more significant quantitative implications than suggested by a study based on time series averages. First, distress costs may at times reach levels much more dramatic than the average simply due to time variation in default rates; second they argue that the necessary risk-adjustment to FDC may

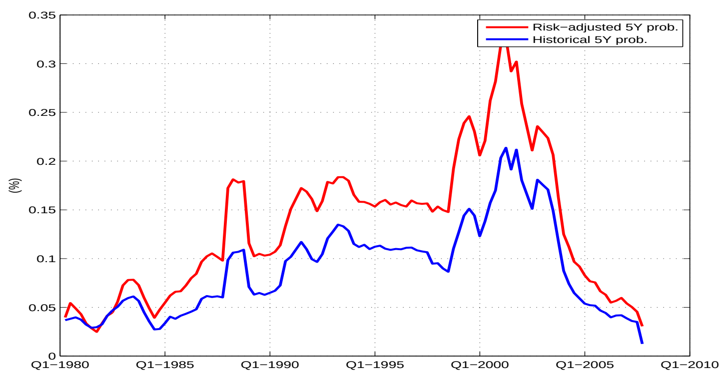

Figure 3: **Default probabilities across time.** The red and blue lines respectively represent the 5-year cumulative default rates with and without risk-adjustment respectively. These default rates are equally weighted averages across our panel of firms at the parameters' sample estimates.

exacerbate this. Our average % FDC based on an ex-post cost of 16.5% is just less than one percent. Over time, however, the FDC is volatile. In the early eighties, it is less than half a percent. As of the late eighties to the end of the nineties, it doubles and remains around 1%. As of the Russian default in late 1998, the FDC begins to trend upwards to reach levels between 1.5% and 1.7%. It can readily increase or decrease by a quarter to a full percentage point from one quarter to the next.

In addition, Figure 4 illustrates the importance of the risk-adjustment highlighted by Almeida and Philippon (2007). On average, there is a significant wedge of around 50 basis points between the historical and risk-adjusted FDC. This wedge has become increasingly important over time. It was almost non-existent in the early eighties, then increased to about 40 basis points and eventually peaked twice in 1998 and 2002/2003 at levels nearing 90 basis points. From then on until the end of our sample, it declined steadily to pre 1998 levels. These patterns are consistent with results documented recently in credit markets. For example, the peak in the third quarter of 2002 and subsequent decline has been reported for default risk premia in credit derivative markets by Berndt et al. (2008). This pattern as well as the peak in 1998 after the Russian default

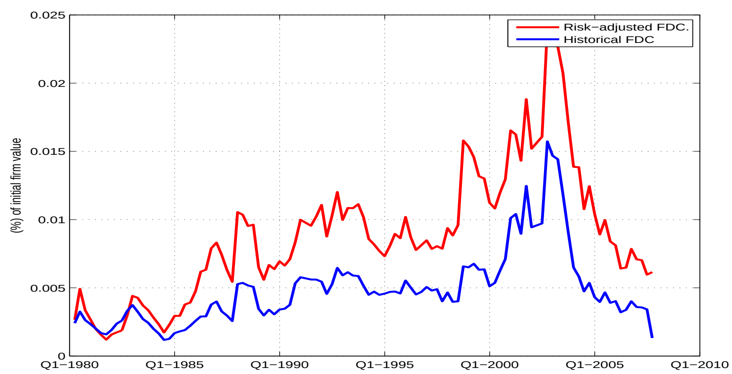

Figure 4: **The risk adjustment of financial distress costs.** The red and blue lines respectively represent the ex-ante financial distress costs with and without risk-adjustment respectively. These financial distress costs are equally weighted averages across our panel of firms at the parameters' sample estimates.

and the LTCM crisis is also present in corporate bond risk premia (see Ericsson and Elkamhi (2008)).

Next, we show that the risk-adjustment to FDC is related to the degree of systematic risk at the firm level. In order to do so, we create quartiles along firms' asset betas. Figure 5 plots the wedge between FDC computed using risk-adjusted and historical term structures of default rates. The blue line is the time series average of the adjustment for the firms in the top beta quartile and the red line for those in the lowest. The two time series behave similarly overall, but the higher beta firms have markedly more volatile FDCs. These firms were also subject to a significantly greater risk-adjustment after the 1987 crash than lower beta firms. The 1998 and 2002 peaks and subsequent decline were experienced by both firm categories.

It is interesting to note that, although very closely related, default costs and default rates are not perfectly correlated. For example, the highest level for the 5-year risk-adjusted default rate does not correspond to the peak in distress costs, highlighting that it is not merely the time variation in probabilities that determines the distress costs. These depend also on the default boundary, which in turn depends on asset volatility. While default rates were at their highest in early 2001, so were asset risk levels. Increased asset risk increases

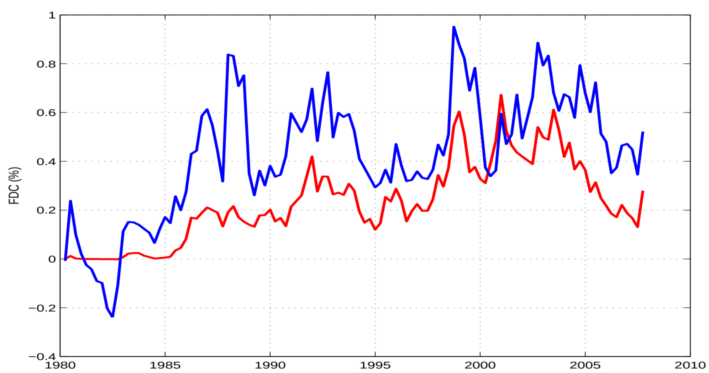

Figure 5: **The risk-adjustment of FDC for different levels of systematic risk.** The lines depict the absolute difference between ex-ante financial distress costs (FDC) computed with and without risk adjustment. The red and blue lines respectively depict the absolute impact of the risk-adjustment of FDC for the firms in the lowest and highest quartiles computed along the mean asset beta across the full sample.

the option value of equity and thus shareholders patience. This is equivalent to lower default boundaries and thus ex-post lower distress costs relative current firm value. Figure 6 illustrates this. The impact of spiking default rates on FDC in 2001 are offset by lower ex-post distress costs ($\alpha \cdot V_B$). As of the time when default rates peak, volatility decrease sharply to pre 2001 levels, leading to a relative increase in the average default boundary. The net effect is that FDC peak when the default boundary ratio levels out while default rates decline.

In addition to addressing the time variation of default rates and financial distress costs, our framework allows us to measure the sensitivities of levered firm value to changes in leverage, as we did in section 3.2. Figure 7 plots the average marginal benefit of an additional dollar in book liabilities (P), $\frac{\partial TS}{\partial P}$ and the marginal cost of an additional dollar of debt, $\frac{\partial FDC}{\partial P}$. On average, with a tax rate of 20% and ex-post distress costs of 16.5%, the benefit of an extra dollar of debt appears to be about twice as large as the offsetting increase in the costs of financial distress - consistent with a significant debt conservatism. However, this is not always the case. In the third quarter of 2002, when the distress costs peak, the sensitivities equate.

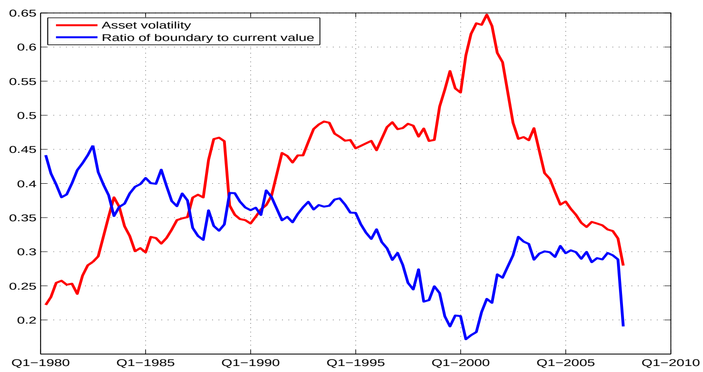

Figure 6: **Volatility and the default boundary.** The red line represents the cross-sectional mean of the unlevered firm level volatility; the blue line graphs the mean of the ratio of default boundary to asset value $\frac{V_B}{V_t}$ computed at each firm quarter. Note that in the Leland and Toft (1996) model, the default boundary is a decreasing function of total asset volatility, in addition to book debt levels, the tax rate, the risk free rate and ex-post distress costs .

5 Robustness and Extensions

So far, our results suggest that most public firms employ too little debt. We reach this conclusion by modeling both the risk-adjusted probability of default, as well as the firm value when default is expected to begin. Combining these with empirical estimates of proportional losses in default, the levels and sensitivities of ex-ante financial distress costs immediately obtains. It is worth reiterating exactly how our procedure differs from previous calculations. If one knows the fraction of the firm expected to be lost due to financial distress, then calculating the dollars lost in default also requires knowledge of the firm's value at default. Because most firms are not currently in financial distress, their current values are much different than the values where financial distress would begin. We use the LT model to forecast such a default boundary, V_B, which we then multiply by proportional ex-post losses, α, to obtain the expected dollars lost to financial distress. The meager financial distress costs we find are not due to differences in risk-adjusted probabilities (see Table 3 in Section 3) or α (we use Andrade and Kaplan's (1998) estimates). Instead, it is allowing for

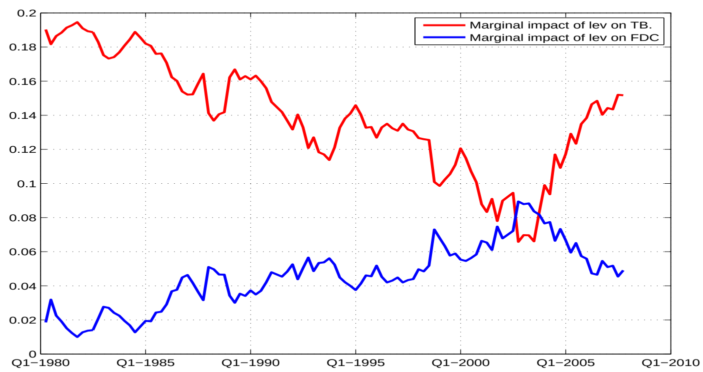

Figure 7: **The marginal benefit of tax shields and costs of financial distress over time**. The blue line represents the average marginal benefit of an additional dollar in book debt (N), $\frac{\partial TS}{\partial N}$. The green line represents the marginal cost of an additional dollar of debt, $\frac{\partial FDC}{\partial N}$. The effective tax rate is assumed to be 15%.

firms to suffer value impairments for reasons other than financial distress, as examining the ratios of $\frac{V_B}{V_t}$ in Table 2 makes clear.

In this section, we extend our results in several dimensions. The first two subsections deal with our assumptions for proportional losses in default, α. First, in 5.1, we explore an issue that follows directly from the sensitivity calculations performed in subsection 3.2. Specifically, instead of specifying proportional default losses from empirical studies, we impute the required losses in default that would allow current capital leverage ratios to be optimal. Subsection 5.2 then demonstrates how to extend the model even further to use bond recoveries *infer* the market's expectations of the firm's expected losses in default (α). Proper estimation requires data not yet available, but we formalize the methodology, and provide an example to provide intuition. Next, in subsection 5.3 we re-estimate the entire model for the subset of firms for whom we can obtain firm-specific tax rates. These estimates allow us to incorporate cross-sectional differences when calculating risk-adjusted tax shields and ex-ante distress costs. Finally, in subsection 5.4, we repeat our calculations under an entirely different specification for default, using Merton's (1974) seminal framework.

DTD	2.5	3.4	4.3	5.3	7.4	All (median)	All (mean)
α^*	0.21	0.33	0.43	0.54	0.73	0.43	0.69
V_t $(MM\$)$	37.6	90.3	158.4	259.0	431.7	139.8	1,992.1
V_B $(MM\$)$	12.7	26.2	36.5	42.2	39.0	27.4	676.0
N	77,865	77,865	77,865	77,865	77,865	389,325	389,325

Table 11: **Proportional ex-post losses (α^*) required to offset tax benefits;** $\tau = 20\%$. This table shows the required levels of ex-post proportional losses, α^*, that equate each firm's sensitivity of distress costs and tax benefits (tax rate=20%). DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. The first five column are quintiles of DTD (means) for the entire sample. V_t is asset value, and the default threshold is V_B. The range reported in Andrade and Kaplan (1998) is 10-23%.

Due to differences between this specification and LT (e.g., the default boundary is exogenous in Merton (1974)) this generates different risk-adjusted probabilities, as well as different firm values at default. However, despite these differences, ex-ante financial distress costs are quantitatively similar to those obtained by the LT model.

5.1 How high do ex-post losses need to be?

Our analysis has so far relied on empirical estimates of ex-post proportional distress losses between 10% and 23%, the range found in Andrade and Kaplan's (1998) study of firms that underwent either leveraged or management buyouts. These estimates have been commonly cited in other studies of ex-ante distress costs including Graham (2000), Molina (2005), and Almeida and Philippon (2007). We have seen that these proportional losses are, in the vast majority of cases, too small to offset the considerable tax benefits of leverage. This conclusion is made even stronger because the sensitivity of tax benefits to leverage is substantially reduced when it is adjusted for systematic risk, as Almeida and Philippon (2007) do for financial distress costs.

Here, we conduct two closely related exercises. First, we freeze the forward-looking tax rate at 20%, and ask what ex-post proportional loss, α, would be required in order for the respective sensitivities to equate. In other words, what α would be needed so that an additional dollar of debt would create exactly as much of a tax shield as it would in ex-ante distress costs? In Table 11, we present the results of this exercise. Given our previous results, the numbers here are unsurprising. As before, we segregate our results by DTD quintile. Even in the riskiest quintile, the median firm would need to lose 21% of its value at default for its observed capital structure to be optimal. This is within Andrade and Kaplans (1998) range, although barely.

DTD	2.6	3.8	4.8	6.1	9.2	All (median)	All (mean)
T^*	0.16	0.10	0.07	0.05	0.02	0.07	0.09
V_t $(MM\$)$	33.2	85.5	166.1	298.3	521.7	141.4	2,049.4
V_B $(MM\$)$	9.0	19.5	31.1	44.0	55.0	24.2	589.3
N	79,490	79,490	79,490	79,490	79,490	397,454	397,454

Table 12: **Marginal tax rates (τ^*) required to offset ex-ante financial distress costs;** $\alpha = 16.5\%$. This table shows the required levels of forward looking marginal tax rates, τ^*, that equate each firm's sensitivity of distress costs and tax benefits (tax rate=20%). DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. The first five column are quintiles of DTD (means) for the entire sample. V_t is asset value, and the default threshold is V_B. Graham's average after-interest effective tax rate in our sample is 19.96% (obtained from his website).

Proceeding across the table, this is no longer the case. Even in the second quintile, proportional losses of one-third are necessary, well outside the range documented by Andrade and Kaplan (1998), and far outside the range of other studies such as Altman (1984) and Warner (1977). Across the entire sample, the median value of α is 0.43, and the mean is 0.69. The skewness is due largely to the fact than in over one-sixth of our sample (17.6%), *complete* proportional losses of 100% are still incapable of reconciling the sensitivities of tax benefits and financial distress.

Next, we conduct the same exercise for tax rates, presented in Table 12. We freeze each firm's proportional ex-post distress loss at 16.5%, and search for the tax rate that would allow the leverage sensitivities of ex-ante financial distress costs and tax benefits to equate. For our sample, the average forward-looking tax rates (after interest) obtained from John Graham's website is 19.96%. However, the table makes clear that effective tax rates far smaller than these provide tax shields large enough to offset ex-ante financial distress costs. The median required tax rate for the riskiest DTD quantile is only 16%, while for the safest quintile is only 2%.

5.2 Inferring ex-post proportional losses from stock prices

All of our ex-ante calculations have used Andrade and Kaplan's (1998) estimates of proportional ex-post default losses. We do this so that we can directly compare our results with previous studies that also use these estimates. Here, we discuss how our basic results can be extended: rather than using empirical ex-post estimates for default losses, these can be inferred directly from the model. Previously, we have required the LT model to match only two moments - the firm's equity volatility and equity value, both of which can be estimated from observables (see the appendix for more details). However, one could do better if he could infer the market's expectation of recovery on the firm's total liabilities, and then search for the value of α that implies such a recovery.[27]

To see this, recall that creditors of the firm's liabilities recover $(1-\alpha)V_B$ dollars on P dollars debt outstanding, for a model implied recovery rate $R^* = \frac{(1-\alpha)V_B}{P}$. Only V_B was estimated in Section 3, as α and P were taken as exogenous, the former from Andrade and Kaplan and the latter from COMPUSTAT. However, if one can specify R^* ex-ante, then a market-implied ex-post loss in default, α^*, emerges. Not only will this estimate of α be firm-specific, but will also track time-series changes in recovery rates, to the extent that these are available. To implement this, one need only extend the two equation system which is standard in the literature on structural models, by including a condition matching the model implied recovery with the input quantity.

Unfortunately, data unavailability prevents this methodology from being practically implemented at this time. The LT model requires the market's expectation of recovery rates on the firm's total liabilities, and because these claims are not traded, we cannot infer the needed information from market prices. One approximation is to look at industry-level average *bond* recoveries from studies such as Altman and Kishore (1996), and use these. However, using recovery rates on bond claims subordinated to the firm's other obligations (pension liabilities, taxes payable, etc.) will understate the overall debt recovery rate. This downward bias in recovery will lead to an upward bias in ex-post proportional distress losses, α.[28] As firm-level data on recoveries becomes more complete, it will be possible to infer not only the likelihood of distress, but also the market's expectation of each firm's losses in distress. This will generate not only more informative estimates of ex-ante distress costs, but will also generate a rich set of empirical implications, e.g., whether high implied α line up with firm characteristics (e.g., asset tangibility, size, etc.) in ways predicted by theory.

[27] See also Chen (2008).

[28] To see this, note that by requiring the model to match a recovery rate that is too low, there will be two effects. First, recalling that recovery $R = \frac{(1-\alpha)V_B}{P}$, for a given default threshold V_B, smaller values of R imply higher values of α. Second, when α increases, we know from our discussion in subsection 3.1 that this makes shareholders less patient. Keeping the firm live is more costly, which increases V_B. Thus, using a lower bound for recovery rates leads to an upper bound for ex-ante financial distress costs.

5.3 Firm-specific tax rates

We obtain a sequence of firm-specific forward-looking marginal effective tax rates from the website of John Graham, and then use these as inputs into the LT model.[29] We are able to match over 85% of the firms in our sample. For each matched firm, we averaged its forward-looking effective tax rate (after interest expense) over all time, and used this for every observation for that firm.[30] The sample average is 19.96%, which is nearly identical to the constant tax rate we use in our main analysis (20%). However, using firm-specific tax rates allows us to capture cross-sectional differences that potentially have an impact on ex-ante distress costs.

To see this, recall that the default boundary, V_B, results from the trade-off faced by equityholders. A higher tax rate, all else equal, increases the expected value of the tax shield, and will make shareholders more patient when making the default decision. As we have already seen, a decrease in V_B has two effects, not only making distress less likely, but also decreasing the loss given distress (equation 2). Because the LT is highly non-linear in a number of parameters including tax rates, modeling these cross-sectional differences could conceivably have meaningful effects.

Table 13 shows the results of this exercise, with proportional losses in distress of 16.5%. Generally, the ex-ante financial distress costs are unaffected by modeling differences in average tax rates. Compared to Table 4 which uses a constant tax rate of 20%, the average percentage FDC is nearly identical (48 bp compared to 45 bp). The sensitivities of ex-ante distress costs to leverage are also similar, as compared to Table 7. On average, they are similar but a bit lower with firm-specifc tax rates (roughly 4 cents per dollar increase in debt compared to 5 cents). The same is true with respect to the tax benefits of leverage. In unreported results, the results are also similar when the Andrade and Kaplan (1998) endpoints, 10% and 23%, are used instead. We conclude from this exercise that our results are robust to cross-sectional difference in forward looking tax rates, although we acknowledge that we have ignored potentially important state and time dependencies as emphasized in Graham (1996).

[29] Firm-specific tax rates are available upon request at http://faculty.fuqua.duke.edu/jgraham.

[30] While this procedure will pick up differences in average tax rates, it does not capture the type of state-dependent considerations considered in Graham (1996). Extending the LT model to allow for state-dependent tax rates is a complex task which lies beyond the scope of this paper. In particular, the default boundary in LT would no longer be time-invariant, which would considerably complicate the present analysis.

DTD	2.4	3.6	4.5	5.8	8.8	All (median)	All (mean)
$\frac{FDC}{V_t}$	0.0210	0.0092	0.0047	0.0021	0.0004	0.0043	0.0091
$\frac{\partial FDC}{\partial P}$	0.074	0.053	0.041	0.031	0.013	0.040	0.050
$\frac{\partial TB}{\partial P}$	0.090	0.129	0.147	0.161.2	0.187	0.149	0.140
V^L $(MM\$)$	37.4	84.8	162.4	281.2	467.3	141.3	2025.2
V_B $(MM\$)$	12.9	20.8	29.3	36.2	36.1	23.8	539.1
N	90,455	90,455	90,455	90,455	90,455	452,274	452,274

Table 13: **Estimates with firm-specific tax rates (Graham),** $\alpha = 0.165$. This table shows the present value of ex-ante financial distress costs implied by the LT model, with firm-specific average tax rates provided from John Graham's website. Unless otherwise specified, all values are medians. DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. The first five column are quintiles of DTD (means) for the entire sample. The first row, $\frac{FDC}{V_t}$, is the present value of ex-ante distress costs capitalized and discounted, as a fraction of current firm value. The second row presents the results of estimating equation (3), but does not filter out the effects of future economic shocks. Row three is the difference between the first two rows. V_t is total firm value, and the default threshold is V_B.

5.4 Specifying default with Merton (1974)

The Leland and Toft (1996) is just one of many structural models that provides the information necessary to ex-ante distinguish between economic shocks and financial distress. We have chosen it because it balances tractability and realistic features of default, as well as matches empirically observed default values (e.g., Davydenko (2007)). However, to provide some insight into the sensitivity of the results to our choice of model, we report the results of re-estimating equation (3), relying on Merton (1974). The Merton model is the canonical structural model and, its limitations and recent extensions notwithstanding, has been used often in the recent literature. Compared to the LT model, there are two main differences. First, in Merton (1974), the firm is required to default if its value drops below the face value of its total liabilities. That is, equity holders do not have the option of servicing equity out of their own pockets, hoping for a turnaround in the firm's fortunes. Second, it is based on a much simpler and less realistic capital structure - the firm is financed exclusively by one issue of discount debt, the non-payment of which is the only trigger for financial distress. The key output of the model is the cumulative risk-adjusted default probability for the debt horizon.

Extensions to multiple default dates necessarily require a somewhat ad hoc implementation. For example, it is tempting to value a stream of cash flows at different dates simply as the value of a portfolio where each cash flow is valued individually using the Merton model.[31] However, such an approach ignores that, at least in the case of financial distress costs, the realization of each cash flow is conditional on the others not taking place - default only occurs once. Computing the appropriate conditional default probabilities is difficult if

[31] See for example Eom, Helwege, and Huang (2004).

DTD	1.43	2.44	3.45	4.72	7.08	All (median)	All (mean)
$\frac{FDC}{V_t}$ with $V_B = TL$	0.027	0.010	0.004	0.001	0.000	0.003	0.009
$\frac{FDC}{V_t}$ with $V_B = V_t$	0.062	0.030	0.014	0.006	0.001	0.012	0.031
Δ	0.035	0.020	0.010	0.005	0.001	0.010	0.022
V_t $(MM\$)$	39.3	87.8	164.2	301.5	528.5	146.7	2130.0
TL $(MM\$)$	20.4	32.7	48.8	68.9	81.3	41.2	874.2
N	90,455	90,455	90,455	90,455	90,455	452,274	452,274

Table 14: **Ex-ante financial distress costs under Merton (1974),** $\alpha = 0.165$. This table shows the present value of ex-ante financial distress costs implied by the Merton (1974) model. Here, the default boundary is total liabilities, TL. Unless otherwise specified, all values are medians. DTD is KMV-Moodys distance to default, defined as $\frac{ln(V_t)-ln(TL)}{\sigma}$. The first five column are quintiles of DTD (means) for the entire sample. The first row, $\frac{FDC}{V_t}$, is the present value of ex-ante distress costs capitalized and discounted, as a fraction of current firm value. The second row presents the results of estimating equation (3), but does not filter out the effects of future economic shocks. Row three is the difference between the first two rows. V_t is total firm value.

default is permitted to occur only at discrete times (in LT it can occur at any instant).[32] Our approach is to approximate, for example, the probability of defaulting in the fifth year conditional on surviving until the end of the fourth as the difference between the cumulative four and five year survival probabilities. These probabilities together with setting $LGD = \alpha \cdot TL$ in equation (3).

For these reasons, we rely mostly on the LT model for our analysis, but here present the Merton (1974) estimates as a benchmark. As before, we present our results by DTD quintiles. As Table 14 shows, the results are quite similar, even with a different specification for default. The FDC estimates using the higher default boundary $V_B = TL$ are quite similar in overall levels to those obtained using the LT model. Since the LGD in the Merton model is higher, this implies that overall default probabilities are lower than in LT. This yields lower estimates of the FDC with the higher default boundary $V_B = V_t$, somewhat lowering the effect of filtering for economic distress. However, given that the LT default probabilities line up quite well with rating-based default rates in Table 3, the importance of this filtering is likely underestimated when using the Merton model.

Overall, this exercise does not suggest that the overall levels of FDC estimated using the LT model are sufficiently sensitive to question the importance of the filtering documented earlier.

[32]Geske (1977) solves this problem and highlights how it quickly becomes computationally intractable for more than a few payment dates.

6 Conclusion

Financial distress and bankruptcy costs are generally thought to be the chief drawbacks to debt finance, countering the substantial tax benefits and possible of managerial agency costs. Indeed, it remains quite puzzling why such highly profitable firms such as Amgen, Intel, and ExxonMobil appear to pay billions in extra taxes that could be shielded by a simple debt-for-equity swap. Under the assumption that target leverage ratios matter for firm values, there are three possibilities.[33] The first is that these firms are in fact leveraged too conservatively, perhaps stemming from entrenched managers who are able to extract private benefits from shareholders by choosing a suboptimally low debt level (e.g., Berger, Ofek, and Yermack (1997)). Alternatively, it is possible that these firms have substantial costs of leverage that we do not recognize, and that these costs offset the tax savings. For example, a growing literature highlights the importance of bond ratings for corporate decisions (e.g., Graham and Harvey (2001), Kisgen (2008), Kayhan and Titman (2008)). A final possibility is that we are incorrectly measuring the financial distress costs we do recognize, as recently emphasized by Molina (2005) and Almeida and Philippon (2007).

Our study addresses the final possibility, and concludes that extreme losses in distress or default - nearly complete destruction of value - are necessary to reconcile the predictions of the trade-off theory with observed leverage ratios. We reach this conclusion by capitalizing financial distress losses that occur when the firm defaults, but in an important departure from previous studies, filter out future economic shocks from financial distress costs. Without such filtering, we find ex-ante distress costs on the order of 5% of current firm value; with the filtering, they are less than 1%. We then formally examine the sensitivities of tax benefits, and compare these to the sensitivities of financial distress costs. Hardly ever (less than 10% of the time) do these sensitivities equate, and when they do, it is for small firms very near default. When "pure" ex-ante financial distress costs are compared to tax benefits, the trade-off theory is far from able to explain the leverage ratios of the large, stable, profitable firms originally singled out by Graham (2000) as being underleveraged.

This again raises the question why the majority of firms appear underleveraged. A significant possibility is that our methodology – and indeed, virtually every ex-ante study of distress costs – misses a large and

[33] We take as given that firms have leverage targets. However, there exists substantial debate as to the importance of target leverage ratios at all. On the one hand, a family of cross-sectional studies (e.g., Bradley, Jarrell, and Kim (1984), Rajan and Zingales (1995), Frank and Goyal (2003)) suggests that leverage ratios are predictable from firm characteristics that proxy for the costs and benefits of leverage. For example, firms with high effective marginal tax rates have high leverage (Graham (1996)), while those tangible assets easily redeployed in bankruptcy choose low leverage (Titman and Wessels (1988)). On the other hand, time series tests have not been as favorable to the trade-off theory, as evidenced by relatively low "speeds of adjustment" toward leverage targets documented by, e.g., Shyam-Sunder and Myers (1999) and Flannery and Rangan (2006). Finally, evidence of market timing (e.g., Baker and Wurgler (2002) or indifference (Welch, 2004) suggests that firms may not adhere to target leverage ratios at all. For a more detailed discussion of these issues, see Parsons and Titman (2008).

important family of distress costs that are incurred prior to default. For example, Opler and Titman (1994) document that excessive leverage can costs firms sales and operating performance, even when they do not default. To the extent that we are missing such pre-default distress costs, our conclusions regarding optimal capital structure are weakened. On the other hand, even when we use very aggressive assumptions for losses in distress (the higher endpoint of Andrade and Kaplan's range of 23%), financial distress costs are unable to offset the tax benefits. Given this, it appears that additional considerations such as managerial preferences are a required component for understanding the cross-section of capital structure.

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Appendix

6.1 Technical details of Leland and Toft (1996) and estimation

A firm has unlevered assets with value V_t. As in Merton (1974), Black and Cox (1976), Brennan and Schwartz (1978) and other papers on structural credit risk models, the dynamics of V_t^U can be described using a continuous diffusion model with constant proportional volatility. The objective and risk adjusted dynamics of the unlevered firm value can be written as

$$
\begin{aligned}
dV_t &= (\mu - \delta)V_t dt + \sigma V_t dW_t, && (5)\\
dV_t &= (r - \delta)V_t dt + \sigma V_t d\tilde{W}_t. && (6)
\end{aligned}
$$

where μ is the expected continuously compounded growth rate for V, r is the risk-free interest rate, and δ is the payout ratio, or proportion of value paid out to security holders. The volatility of the unlevered assets is σ, and W_t and $\tilde{W}_t$ are Brownian motions under objective and risk-adjusted probabilities respectively.

Additionally, the firm has debt B_t, which generates both a tax shield (TS) and ex-ante financial distress costs (FDC). In the LT model the firm operates with a stationary debt structure. This debt structure consists of a continuum of bonds with maturities ranging from immediate to an upper bound T, which corresponds to the maturity of newly issued debt. As a bond matures it is replaced by a new bond with the same coupon, initial maturity and principal. Should the fortunes of a firm decline, the amount raised by reselling a nominally unchanged bond must clearly decrease. The difference between the amount raised and that necessary to withdraw the old bond is funded by shareholders should they find it optimal to keep the firm afloat. New bonds are issued and old bonds retired at a rate of $p = (\frac{P}{T})$ per year, where P is the total principal value of all outstanding bonds. This condition ensures that as long as the firm remains solvent, at any time during the life of the firm, the total outstanding debt principal will be invariant and equal to P. These latter assumptions also ensure that the total coupon C paid for all outstanding bonds is also stationary.

Levered firm value, V^L, is equal to the value of its unlevered assets (V_t^U) plus the tax shield (TS) less the costs of financial distress (FDC), or

$$
\begin{aligned}
V_t^L &= V_t + TS(V_t) - FDC(V_t)\\
&= E(V_t) + D(V_t) && (7)
\end{aligned}
$$

In equation (7), we are primarily interested in quantifying the ex-ante distress costs FDC. In our estimation, we will exploit that there is a closed form solution for equity in order to estimate asset values, V_t^U, and their volatility, σ, the key unobservable inputs to the analytical expressions for the components of levered firm value.

The value of the tax shield is given by

$$
TS(V_t) = \frac{\tau C}{r} \cdot \left(1 - \left(\frac{V_t}{V_B}\right)^x\right),
$$

and the ex-ante distress costs are given by

$$FDC(V_t) = LGD \cdot \left(\frac{V_t}{V_B}\right)^x,$$

where LGD is given by $\alpha \cdot V_B$.

The value of debt is given by

$$D(v_t) = \frac{C}{r} + \left(N - \frac{C}{r}\right)\left(\frac{1 - e^{-rM}}{rM} - I(v_t)\right) + \left((1-\alpha)\,L - \frac{C}{r}\right) J(v_t),$$

The default boundary is given by

$$V_B = \frac{\frac{C}{r}\left(\frac{A}{rM} - B\right) - \frac{AP}{rM} - \frac{\tau C x}{r}}{1 + \alpha x - (1-\alpha)B}$$

where

$$\begin{aligned} A = {} & 2ye^{-rM}\phi\left[y\sigma\sqrt{M}\right] - 2z\phi\left[z\sigma\sqrt{M}\right] \\ & - \tfrac{2}{\sigma\sqrt{M}} n\left[z\sigma\sqrt{M}\right] + \tfrac{2e^{-r\Upsilon}}{\sigma\sqrt{M}} n\left[y\sigma\sqrt{M}\right] + (z-y) \end{aligned}$$

$$B = -\left(2z + \frac{2}{z\sigma^2 M}\right)\phi\left[z\sigma\sqrt{M}\right] - \frac{2}{\sigma\sqrt{M}} n\left[z\sigma\sqrt{M}\right] + (z-y) + \frac{1}{z\sigma^2 M},$$

and $n[\cdot]$ denotes the standard normal density function.

The components of the debt formulae are

$$I(v) = \frac{1}{r\Upsilon}\left(i(v) - e^{-r\Upsilon} j(v)\right)$$

$$i(v) = \phi[h_1] + \left(\frac{v}{L}\right)^{-2a}\phi[h_2],$$

$$j(v) = \left(\frac{v}{L}\right)^{-y+z}\phi[q_1] + \left(\frac{v}{L}\right)^{-y-z}\phi[q_2]$$

and

$$J(v) = \frac{1}{z\sigma\sqrt{M}}\begin{pmatrix} -\left(\frac{v}{L}\right)^{-a+z}\phi[q_1]\,q_1 \\ +\left(\frac{v}{L}\right)^{-a-z}\phi[q_2]\,q_2 \end{pmatrix}$$

. Finally,

$$q_1 = \frac{-b - z\sigma^2 M}{\sigma\sqrt{M}}$$

$$q_2 = \frac{-b + z\sigma^2 M}{\sigma\sqrt{M}}$$

$$h_1 = \frac{-b - y\sigma^2 M}{\sigma\sqrt{M}}$$

$$h_2 = \frac{-b + y\sigma^2 M}{\sigma\sqrt{M}}$$

and

$$y = \frac{r - \beta - 0.5\sigma^2}{\sigma^2}$$
$$z = \frac{\sqrt{y^2\sigma^4 + 2r\sigma^2}}{\sigma^2}$$
$$x = y + z$$
$$b = \ln\left(\frac{v}{L}\right).$$

As a result, equity can be valued simply as

$$E(V_t) = V_t + TS(V_t) - BC(B_t) - D(V_t). \tag{8}$$

6.2 Estimation

The advantage of a structural model such as LT is that it simultaneously provides closed form expressions for FDC, default probabilities, equity values and volatilities. The two key inputs to the model, asset value V_t and volatility σ cannot be observed. Instead they are inferred by requiring the model to fit observables. We implement the model by for each firm quarter matching the model implied total equity value and equity volatility with the observed market capitalization and trailing historical volatility. This procedure is widely used in industry and academic research.[34] Prior to this procedure, a number of additional model parameters need to be determined.

- Payout rate (δ). This parameter determines how much cash flow is available at each point in time to service debt and pay dividends. One way to estimate it would be to write $\delta = \frac{IE+DIV}{V_t} = \frac{IE}{TL}lev + \frac{DIV}{E}(1 - lev)$, where IE denotes interest expenses, DIV dollar dividends paid by the firm, TL total liabilities and $lev = \frac{TL}{TL+E} = \frac{TL}{V}$. This can be done as long as we are willing (in this step) to proxy V_t by the sum of the market capitalization and total liabilities. We carry out this calculation for the subset of firms for which we have the required data. We then take the average of 2.9% and impose it across all firms. This number is in the range of estimates provided in recent work on payout rates by Larrain and Yogo (2008).
- Debt structure (C, P, T). We use total liabilities to proxy for P, the total principal of debt, and assume for simplicity that the aggregate coupon rate on debt to be the risk free rate, and thus $C = r \cdot N$. Although this may seem a strong assumption, note that many forms of debt do not pay interest and that the rates on the others depend on the maturity. An alternative used in the literature is to assume that C can be proxied by the coupon rate on corporate debt. This is appealing as it would link the coupon rate to market data. However, it will tend to bias the rate upwards as corporate bonds tend to be issued as long term instruments and term structures of credit spreads tend to be upward sloping. In our case this is not a viable approach as using bond data would greatly limit the size of our data set. An alternative approach which is also limited by data considerations would be to use actual figures for interest expenses. For the subset of our data for which this is available we find that the ratio of interest expense to total liabilities is about 5.2% annually. The average risk free rate use in the estimation is 6.6%. There is limited data available for debt maturity. We use an average of 6.76 years which corresponds to a value of $T = 3.38$ for the maturity of newly issued debt in the Leland and Toft (1996) model.[35]

[34] See for example Jones, Mason and Rosenfeld (1984, 1985) and Bharath and Shumway (2008).

[35] This number is taken from an empirical study by Stohs and Mauer (1994).

- Risk free rate. We use the 5 year Treasury rate.
- *Ex post* distress costs (α). For this parameter we use the upper and lower bounds reported in Andrade and Kaplan (1998) 23% and 10% as well as the midpoint, 16.5%.
- Corporate tax rate. In the bulk of our analysis we use 20% for all firm quarters. As a robustness exercise, we consider average firm level tax rates obtained from John Graham's web site in section 5.3.

Once these have been determined, we rely on the following two equations:

$$\begin{aligned} E_{mkt} &= E_{mod}(V_t; \sigma) \\ \sigma_E &= \frac{V_t}{E_{mod}(V_t; \sigma))} \frac{\partial E_{mod}(V_t; \sigma)}{\partial V_t} \sigma \end{aligned}$$

where subscripts *mod* and *mkt* denote model and market values respectively. Estimates of value and volatility $\left(\hat{V}_t, \hat{\sigma}\right)$ are obtained by a straightforward numerical solution to the two above equations.

6.3 Inferring α from the LT model

If reliable data for *debt* recoveries are available, the approach above can be extended to compute market implied levels for the ex-post distress costs. Note that the recovery for total debt is given by

$$R = \frac{(1-\alpha) \cdot V_B}{P}$$

Given that equity value is a function of the default boundary V_B which in turn is a function of α and σ, this equation cannot, for a given R, be solved separately for α. Instead the equation system above needs to be expanded as follows:

$$\begin{aligned} E_{mkt} &= E_{mod}(V_t; \sigma, \alpha) \\ \sigma_E &= \frac{V_t}{E_{mod}(V_t; \sigma, \alpha))} \frac{\partial E_{mod}(V_t; \sigma, \alpha)}{\partial V_t} \\ R_{mkt} &= \frac{(1-\alpha) \cdot V_B(\sigma, \alpha)}{P} \end{aligned}$$

These three equations can then be solved jointly for asset value, volatility and the loss rate α.