Crises and Liquidity in Over-the-Counter Markets*

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Abstract

We study the efficiency of dealers' liquidity provision and the desirability of policy intervention in over-the-counter (OTC) markets during crises. Our theory emphasizes two key frictions in OTC markets: finding counterparties takes time, and trade is bilateral, with quantities and prices determined by bargaining. We model a crisis as a negative shock to investors' asset demands that lasts until a random recovery time. In this context, dealers can provide liquidity to outside investors by acting as counterparties in trades and by accumulating asset inventories. We find that, when frictions are severe, even well capitalized dealers may not find it optimal to accumulate inventories, given that investors choose asset positions that require small reallocations. In such circumstances, welfare can increase if the government steps in, purchases private assets on its own account, and resells them when the economy recovers.

Keywords: liquidity, asset inventories, execution delays, search, bargaining

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1 Introduction

Many of the financial instruments at the core of the recent financial crisis — mortgage-backed securities, collateralized debt obligations, credit default swaps— are traded in over-the-counter markets (OTC), outside of organized exchanges. Liquidity in these markets is provided on a voluntary basis by broker-dealers such as large investment banks, who match buyers and sellers and, in the event of large selling pressures, typically buy assets on their own account. While dealers' liquidity provision seems inconspicuous in normal times, it has proved inadequate during the crisis (see, e.g., Brunnermeier, 2009, and Bank for International Settlements, 2009). The ensuing liquidity disruptions prompted the Federal Reserve to undertake unprecedented policy actions: it offered primary dealers the opportunity to borrow capital cheaply through various lending facilities and, in some markets, it purchased assets on its own account.[1]

We study the efficiency of dealers liquidity provision and the desirability of policy intervention in over-the-counter (OTC) markets during times of crisis. We focus on the role played by two characteristic trading frictions of OTC markets, search and bargaining, and abstract from credit-market frictions that may limit dealers' access to capital.[2] There is little doubt that credit frictions are important for explaining why the Federal Reserve wants to ease dealers' access to capital during crises. However, they don't easily explain why the Federal Reserve should also buy assets on its own account, let alone advocate changing the structure of OTC markets (Bernanke, 2009b). Our analysis shows that, when trading frictions are large, well-capitalized dealers may be unwilling to provide liquidity, and even under circumstances when it would be socially efficient for them to do so. Since the inefficiencies are due to trading frictions, supplying more capital to dealers would be ineffective: dealers would hoard the injected capital instead of providing liquidity. By contrast, we find that social welfare can increase if the government acts as a "liquidity provider of last resort" by purchasing assets on its own account in order to resell them when the economy recovers. In the long-run, when the government can implement policies that affect the structure of OTC markets, our analysis suggests that it should aim at reducing dealers' market power.

Our work builds on the search-theoretic models of Duffie, Gârleanu, and Pedersen (2005),

[1] In March 2008 the Federal Reserve introduced the Term Securities Lending Facility, an auction facility that allows primary dealers (investment banks, broker-dealers) to borrow Treasury securities for long periods against less liquid collateral. The Federal Reserve also introduced the Primary Dealer Credit Facility, an overnight loan facility that provides funding to primary dealers (see Fleming, Hrung, and Keane, 2009). Lastly, on March 2009, it announced it would purchase up to $1.25 trillion in Mortgage Based Securities (see Bernanke, 2009a).

[2] Credit market imperfections have been analyzed in earlier work, for instance, Gromb and Vayanos (2002), Weill (2007), and Brunnermeier and Pedersen (2009).

Weill (2007), and Lagos and Rocheteau (2007, 2009). Outside investors cannot trade continuously in a Walrasian market. Instead, they receive infrequent and random trading opportunities with dealers, who are able to trade continuously with each other. This search friction provides a natural description of bilateral trades in OTC markets, and it also captures a wide range of impediments that make it more difficult to trade financial assets during crises, such as disruptions in communication systems, or outright dealer failures, such as that of Lehman Brothers in September 2008. To create a crisis, we hit our theoretical OTC market with an aggregate negative shock that reduces investors' willingness to hold the asset. The crisis state persists until some random time at which investors receive the opposite shock and the economy recovers.

We determine the conditions under which well-capitalized, profit-maximizing dealers provide liquidity to outside investors, accumulating assets in their inventory during the crisis and unloading these assets when the economy recovers. We find that the amount of liquidity provided by dealers varies nonmonotonically with the magnitude of the trading frictions. More precisely, consider a spectrum of OTC markets ranging from those with very small frictions, for instance markets for Treasury securities or wholesale foreign exchange, to those with large trading frictions, such as some markets for subprime mortgage-backed securities. We find that dealers provide no liquidity in markets at either end of the spectrum and some liquidity in markets lying in the the middle of the spectrum. In particular, when trading frictions are very large, investors become reluctant to hold extreme asset positions because they anticipate that these positions will be very difficult to unwind. All investors end up with a similar "average" asset position and therefore do not demand much liquidity from dealers. Because of this accommodation in liquidity demand, dealers do not provide any liquidity in equilibrium.

In markets where dealers have a large degree of bargaining power, the lack of liquidity provision is socially inefficient, given the search frictions. This finding has three main policy implications. First, since dealers in our analysis have unrestricted access to capital, it immediately implies that injecting capital is ineffective: dealers would hoard it instead of using it to purchase assets. Second, we show that if the government acts as a "liquidity-provider-of-last-resort," i.e., if it purchases assets during the crisis in order to resell them when the economy recovers, equilibrium social welfare can increase. The third implication is that, if the policymaker is able to implement policies that affect the structure of the market, it should aim at reducing the market power of dealers.

Related literature

Our work belongs to the recent literature that studies search and bargaining frictions in asset markets, and pursues the inventory-theoretic approach to dealership markets that goes back to Stoll (1978), Ho and Stoll (1983) and Amihud and Mendelson (1980), as well as the recent work of Hendershott and Menkveld (2009). We go beyond previous studies by working out the out-of-steady-state dynamics induced by aggregate shocks while allowing *both* dealers and investors to hold unrestricted asset positions. Lagos and Rocheteau (2007, 2009) relax Duffie, Gârleanu, and Pedersen's (2005) restriction that investors can hold 0 or 1 unit of the asset but maintain the assumption that dealers cannot hold asset inventories, so their model remains silent about the desirability of liquidity provision by dealers in the face of temporary selling pressures. Weill (2007) allows dealers to hold unrestricted inventories but maintains the 0-1 restrictions on investors' asset holdings. In this paper we relax the holding restrictions of both dealers and investors, and show that the endogenous response of investors' holdings to trading frictions is a key determinant of dealers' equilibrium liquidity provision. For instance, in contrast with Weill (2007), we show that dealers may not find it in their interest to provide liquidity during the crisis when it would be socially optimal for them to do so. Weill (2007) also assumed that, after the initial shock, the recovery path was deterministic. Instead, we consider a more realistic setup in which the recovery is a random event, which generates the new implication that rational dealers find it optimal to buy assets while the market price continues to decline, and re-sell them while the market price continues to go up (the associated price divergence is similar to that of Kondor, 2009, except that it is based entirely on search frictions instead of capital market frictions). Lastly, our model can provide a rationale for purchases of assets by the government in OTC markets during crisis. In independent work, Chiu and Koeppl (2009) analyze welfare-improving purchases of "lemons" by the government, in a search model with adverse selection.

There is a related literature on liquidity provision by dealers, e.g., the seminal model of Grossman and Miller (1988), where competitive dealers provide liquidity in order to share risk with investors. Recent work in that tradition includes Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009), who study the impact of borrowing constraints on the supply of liquidity, and Huang and Wang (2009), who endogenize the supply and demand of liquidity via participation costs. In contrast to this line of work, our dealers are not competitive and do not share risk with investors. Instead, they have market power and, as

Demsetz (1968) emphasized, they provide immediacy: they speed up the allocation of assets to their final holders. Bernardo and Welch (2004) study how dealers provide liquidity during a financial-market run, in a model in the tradition of Diamond and Dybvig (1983). Lastly, our study of government liquidity provision is related to some results in the payment-system literature, most notably Freeman (1996, 1999), who shows that a temporary government purchase of private IOUs may improve welfare in the presence of settlement frictions.

2 The environment

Time is continuous, runs forever, and is indexed by $t \geq 0$. There is one asset and one perishable good, which we use as a numéraire. The asset is durable, perfectly divisible, and in fixed supply, $A > 0$. The numéraire is produced and consumed by all agents. The economy is composed of two types of infinitely-lived agents who discount the future at the same rate, $r > 0$, a unit measure of investors, and a unit measure of dealers.

The instantaneous utility function of an investor is $\zeta(t)u_i(a) + c$, where $a \geq 0$ represents the investor's asset holdings, c is the net consumption of the numéraire good ($c < 0$ if the investor produces more than he consumes), $i \in \{1, ..., I\}$ indexes an idiosyncratic preference shock, and $\zeta(t)$ represents an aggregate preference shock. The utility function $u_i(a)$ is strictly increasing, concave, continuously differentiable, and satisfies $u_i'(0) = \infty$. We also assume that $u(a)$ is either bounded below or above. Investors receive idiosyncratic preference shocks that occur at Poisson arrival times with intensity $\delta > 0$. When the preference shock hits, the investor draws preference type i with probability π_i. These preference shocks capture the notion that investors value the services provided by the asset differently over time, and they generate a need for them to periodically change their asset holdings.[3] At time zero the distribution of investors across the preference types $\{1, ..., I\}$ is at its steady state, $\{\pi_i\}_{i=1}^{I}$.

We trigger a financial crisis with an aggregate preference shock. As illustrated in Figure 1, we assume that $\zeta(t) = \theta < 1$ for all $t \in [0, T_\rho)$, and $\zeta(t) = 1$ for all $t \geq T_\rho$, where T_ρ is an exponentially distributed random variable with mean $1/\rho$, independent from everything else.[4]

[3] As in Duffie, Gârleanu, and Pedersen (2005), our preference specification associates a certain utility to the investor as a function of his asset holdings. The utility the investor gets from holding a given asset position could be simply the value from enjoying the asset itself, as would be the case for real assets such as houses or durables. In the context of financial markets, one should view $u_i(a)$ as a reduced-form utility function that stands in for the various reasons why investors may want to hold different quantities of the asset: differences in liquidity needs, financing or financial-distress costs, correlation of asset returns with endowments (hedging needs), or relative tax disadvantages. By now, several papers have formalized the "hedging needs"interpretation. Examples include Duffie, Gârleanu, and Pedersen (2007), Vayanos and Weill (2008), and Gârleanu (2009).

[4] Although we follow the spirit of Grossman and Miller (1988), we depart from their model in two ways.

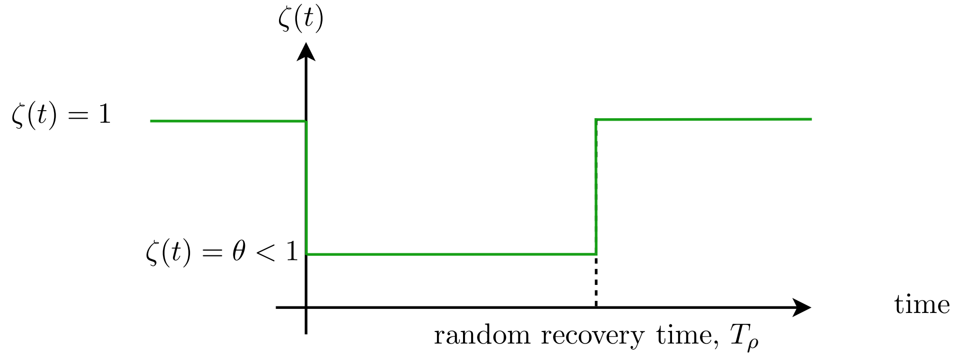

Figure 1: The aggregate preference shock

A small θ indicates that the crisis is severe, and a small ρ that it is expected to be long-lived. Although assuming a preference shock is admittedly a reduced-form model of a crisis, it is in the spirit of the aggregate endowment shocks that are commonly used in the literature (see, e.g., Grossman and Miller, 1988). It also admits several reasonable interpretations: a shock to the riskiness (or "toxicity") of the asset, a "flight to liquidity" (Longstaff, 2004), or a sudden need for cash (Diamond and Dybvig, 1983).

To capture the intuitive notion that dealers are not the final holders of the asset, we assume that their instantaneous utility is c, i.e., that they derive no direct utility from holding the asset. We assume that dealers can continuously buy and sell the asset in an interdealer market, at price $p(t)$. Investors, on the other hand, can only trade periodically and through a dealer. Specifically, we assume that investors contact a randomly chosen dealer at Poisson arrival times with intensity $\alpha > 0$. Once the investor and the dealer have made contact, they negotiate the quantity of assets that the dealer will acquire (or sell) in the interdealer market on behalf of the investor and the intermediation fee that the investor will pay the dealer for his services. After completing the transaction, the dealer and the investor part ways. The trading arrangement is illustrated in Figure 2.

First, in our model, the length of the crisis is stochastic, so dealers' uncertainty about the recovery will influence their incentive to provide liquidity. Second, in Grossman and Miller, dealers provide liquidity in order to share risk with outside investors, while in the present model, dealers have no such direct utility motive for holding assets. Dealers indirectly derive value from holding the asset because they are continuously present in the market, so they can "time the market" better than outside investors. This leads them to hold inventories and, in the aggregate, speeds up the allocation of assets to their final holders.

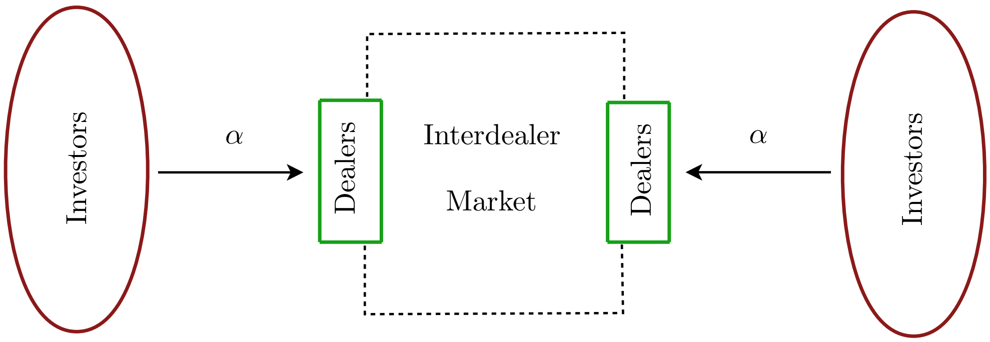

Figure 2: Trading arrangement

3 Equilibrium

We characterize an equilibrium in two steps: we first solve for the equilibrium after the recovery time, for every possible T_ρ. Then, we solve for the equilibrium during the crisis, before T_ρ has been realized.

3.1 The path to recovery

In this section we describe the path of the economy following T_ρ. The aggregate preference shock is $\zeta(t) = 1$ for all $t \geq T_\rho$. We take as given two initial conditions: the realization of T_ρ, and the dealers' inventories at the time where the economy recovers, $A_d(T_\rho)$. To simplify notations in what follows, we suppress the explicit dependence of endogenous variables on these two initial conditions.

3.1.1 The terms of trades in bilateral meetings

Consider a meeting at time $t \geq T_\rho$ between a dealer who is holding inventory a_d and an investor of type i who is holding inventory a. Let a' denote the investor's post-trade asset holding and ϕ be the intermediation fee.[5] The pair (a', ϕ) is taken to be the Nash solution of a bargaining problem in which the dealer has bargaining power $\eta \in [0, 1]$. Let $V_i(a, t)$ denote the value (maximum attainable expected discounted utility) of an investor with preference type i who is holding a quantity of asset a at time $t \geq T_\rho$. The investor's gain from trade is

$$V_i(a', t) - V_i(a, t) - p(t)(a' - a) - \phi.$$

[5] In our formulation we assume that the investor pays the dealer a fee. However, the bargaining problem can be readily reinterpreted as one in which the dealer pays the investor a bid price that is lower than the market price if the investor wants to sell, and charges an ask price that is higher than the market price if the investor wants to buy. See Lagos and Rocheteau (2009) for details.

Analogously, let $W(a_d, t)$ denote the value of a dealer who is holding inventory a_d at time $t \geq T_\rho$. Then, the utility of the dealer is $W(a_d, t) + \phi$ if an agreement (a', ϕ) is reached and $W(a_d, t)$ in case of disagreement, so the dealer's gain from trade is equal to the fee, ϕ.[6] The outcome of the bargaining is given by

$$[a_i(t), \phi_i(a,t)] = \arg\max_{(a',\phi)} [V_i(a',t) - V_i(a,t) - p(t)(a'-a) - \phi]^{1-\eta}\phi^{\eta}.$$

Hence, the investor's new asset holding solves

$$a_i(t) = \arg\max_{a'} \left[V_i(a',t) - p(t)a'\right], \tag{1}$$

and the intermediation fee is

$$\phi_i(a,t) = \eta \left\{V_i\left[a_i(t), t\right] - V_i(a,t) - p(t)\left[a_i(t) - a\right]\right\}. \tag{2}$$

According to (1), the investor's post-trade asset holding is the one he would have chosen if he were trading in the asset market himself, rather than through a dealer. According to (2), the intermediation fee is set so as to give the dealer a share η of the gains associated with readjusting the investor's asset holdings.[7]

3.1.2 The dealer's problem

The value function of a representative dealer who is holding asset position a at time $t \geq T_\rho$ solves

$$W(a,t) = \max_{q(s)} \left\{-\int_t^{\infty} e^{-r(s-t)} p(s)q(s)ds\right\} + \Phi(t), \tag{3}$$

subject to the law of motion, $\dot{a}_d(s) = q(s)$, the short-selling constraint $a_d(s) \geq 0$, and the initial condition, $a_d(t) = a$. Here, $a_d(s)$ represents the stock of assets that the dealer is holding and $q(s)$ is the quantity that he trades for his own account at time s. The dealer gets utility $-p(s)q(s)$ from changing his inventory. The function $\Phi(t)$ is the expected present discounted value of future intermediation fees from time t onward which, by (2), is indepen-

[6]The outcome of the bilateral trade does not affect the dealer's continuation payoff, $W(a_d, t)$, because he has continuous access to the asset market and his trades are executed instantaneously. The dealer may fill an investor's order partially or in full by trading out of, or for his own inventory of the asset. A dealer following an optimal plan must be indifferent, when executing a trade, between using his inventories or not because he has continuous access to the asset market and all the transactions he is involved in are instantaneous.

[7]Our choice of notation for the bargaining solution in (1) and (2) emphasizes the fact that the terms of trade depend on the investor's preference type but are independent of the dealer's inventories. In addition, the investor's post-trade asset holding is independent of his pretrade holding, while the intermediation fee is not.

dent of the dealer's asset holdings. This formulation makes it clear that dealers trade assets in two ways: continuously, in the competitive market, or sporadically at random times, in bilateral negotiations with investors. Since dealers have linear preferences and they can trade instantaneously and continuously in the competitive interdealer market, their optimal choice of asset holdings is independent of what happens in bilateral negotiations with investors. The following lemma describes the solution to the dealer's inventory accumulation problem:

Lemma 1. *Suppose that the price path, $p(s)$, is differentiable and satisfies the no-bubble condition,* $\lim_{s\to\infty} e^{-rs}p(s) = 0$. *Then, a bounded inventory path, $a_d(s)$, with initial condition $a_d(t) = a$ solves the dealer's problem, (3), if and only if for all $s > t$:*

$$\dot{p}(s) - rp(s) \leq 0, \quad \text{with equality if} \quad a_d(s) > 0. \tag{4}$$

Several comments are in order. First, the assumption of differentiability and the no-bubble condition are only made to simplify the exposition: in Lagos, Rocheteau, and Weill (2007) we show that these two conditions must, in fact, hold in any equilibrium. Second, the lemma restricts attention to bounded inventory paths because this property must also hold in equilibrium. Indeed, a group of agents can hold an unbounded positive position only if some other group holds the opposite negative one, which is ruled out by the short-selling constraint. Then, the "only if" part of the lemma provides restrictions on the equilibrium price path, given any bounded solution $a_d(t)$ of the dealer's problem. The "if" part of the Lemma is a standard sufficient condition for "speculator" optimality: a dealer holds positive inventory if the flow cost of buying the asset, $rp(s)$, is equal the the capital gain, $\dot{p}(s)$, and he holds no inventory if it is smaller.

3.1.3 The investor's problem

We now proceed with an analysis of the investor's problem. The value function corresponding to an investor with preference type i who is holding a assets at time $t \geq T_\rho$, $V_i(a,t)$, satisfies

$$\begin{aligned} V_i(a,t) \;\; = \;\; & \mathbb{E}_i\bigg[\int_t^T e^{-r(s-t)} u_{k(s)}(a) ds + \\ & e^{-r(T-t)}\{V_{k(T)}[a_{k(T)}(T),T] - p(T)[a_{k(T)}(T) - a] - \phi_{k(T)}(a,T)\}\bigg], \end{aligned} \tag{5}$$

where T denotes the next time the investor meets a dealer, and $k(s) \in \{1, ..., I\}$ denotes the investor's preference type at time s. The expectations operator, $\mathbb{E}_i[\,\cdot\,]$, is taken with respect to the random variables T and $k(s)$ and is indexed by i to indicate that the expectation is conditional on $k(t) = i$. Over the interval of time $[t, T]$ the investor holds a units of the asset and enjoys the discounted sum of the utility flows associated with this holding of a (the first term on the right side of (5)). The length of this time interval, $T - t$, is an exponentially distributed random variable with mean $1/\alpha$. The flow utility is indexed by the preference type of the investor, $k(s)$, which follows a compound Poisson process. At time T the investor contacts a random dealer and readjusts his holdings from a to $a_{k(T)}(T)$. In this event the dealer purchases a quantity $a_{k(T)}(T) - a$ of the asset in the market (or sells if this quantity is negative) at price $p(T)$ on behalf of the investor, and the investor pays the dealer an intermediation fee, $\phi_{k(T)}(a, T)$. Both the fee and the asset price are expressed in terms of the numéraire good.

Substituting the terms of trade (1) and (2) into (5), it is apparent that, from the investor's standpoint, the stochastic trading process and the bargaining solution are payoff-equivalent to an alternative trading mechanism in which the investor has all the bargaining power in bilateral negotiations with dealers, but he only gets to meet dealers according to a Poisson process with arrival rate $\kappa \equiv \alpha(1 - \eta)$. Consequently, we can rewrite (5) as

$$V_i(a,t) = \mathbb{E}_i\left[\int_t^{\tilde{T}} u_{k(s)}(a)\, e^{-r(s-t)}\, ds + e^{-r(\tilde{T}-t)}\{p(\tilde{T})a + \max_{a'}[V_{k(\tilde{T})}(a',\tilde{T}) - p(\tilde{T})a']\}\right], \quad (6)$$

where the expectations operator, $\mathbb{E}_i$, is now taken with respect to the random variables $\tilde{T}$ and $k(s)$, where $\tilde{T} - t$ is exponentially distributed with mean $1/\kappa$.

After subtracting $p(t)a$ from (6) and ignoring all the terms that do not depend on the asset holding a, we find that the problem of an investor with preference shock i, who gains access to the market at time t, consists of choosing $a \geq 0$ in order to maximize

$$\mathbb{E}_i\left[\int_t^{\tilde{T}} u_{k(s)}(a) e^{-r(s-t)}\, ds - \left(p(t) - e^{-r(\tilde{T}-t)} p(\tilde{T})\right) a\right]. \quad (7)$$

Intuitively, the investor chooses his asset holdings in order to maximize the present value of his utility flow net of the cost of purchasing the asset at time t and reselling it at the next time $\tilde{T}$ when he can readjust his holdings. The next lemma offers a simpler, equivalent formulation of the investor's problem.

Lemma 2. *Let*

$$U_i(a) = \frac{(r+\kappa)\, u_i(a) + \delta \sum_{j=1}^{I} \pi_j u_j(a)}{r+\kappa+\delta} \tag{8}$$

$$\xi(t) = (r+\kappa)\left(p(t) - \int_0^{\infty} \kappa e^{-(r+\kappa)s} p(t+s)ds\right), \tag{9}$$

assume that $p(t)e^{-rt}$ is decreasing and satisfies the no-bubble condition. Then, a bounded process $a(t)$ solves the investor's problem if and only if, when the investor contacts the market with current type i,

$$a(t) = a_i(t) \text{ where } U_i'\left[a_i(t)\right] = \xi(t). \tag{10}$$

The assumption that $p(t)e^{-rt}$ is decreasing is without loss of generality, because it will be true in an equilibrium: otherwise if there were two times $t_1 < t_2$ such that $p(t_1) < e^{-r(t_2-t_1)}p(t_2)$, then a dealer could make unbounded profit by purchasing at t_1 and reselling at t_2.

Intuitively, $U_i(a)$ is the expected flow of utility that the investor enjoys from holding a units of the asset until his next opportunity to rebalance his holdings, and $\xi(t)$ is an investor's effective cost of holding the asset during the intercontact time period: the purchasing price minus the expected discounted resale value of the asset, all expressed in flow terms. Differentiating (9), we can expresse the relationship between $\xi(t)$ and $p(t)$ as

$$\dot{p}(t) - rp(t) = \frac{\dot{\xi}(t)}{r+\kappa} - \xi(t). \tag{11}$$

From (11), the dealer's first-order condition, (4), can be rewritten as

$$\dot{\xi}(t) - (r+\kappa)\xi(t) \leq 0, \quad \text{with an equality if } a_d(t) > 0. \tag{12}$$

Equations (10) and (12) illustrate the main differences between dealers and investors in our setup. Relative to investors, dealers enjoy no direct utility from holding the asset, but they get an extra return captured by $\dot{\xi}(t)/(r+\kappa)$. This reflects a dealer's ability to make capital gains by exploiting his continuous access to the asset market.

3.1.4 The equilibrium path during the recovery

Given the solutions to the investors' and dealers' problems, we are now ready to study the determination of the asset price. Since each investor faces the same probability of accessing the market irrespective of his asset holdings, and since these probabilities are independent across investors, we appeal to the law of large numbers to assert that the flow supply of assets by investors is $\alpha [A - A_d(t)]$, where $A_d(t)$ is the aggregate stock of assets held by dealers. The measure of investors with preference shock i who are trading in the market at time t is $\alpha\pi_i$, where π_i is the ergodic measure of investors with preference type i. Therefore, the investors' aggregate demand for the asset is $\alpha \sum_{i=1}^{I} \pi_i a_i(t)$, and the net supply of assets by investors is $\alpha[A - A_d(t) - \sum_{i=1}^{I} \pi_i a_i(t)]$. The net demand from dealers is $\dot{A}_d(t)$, the change in their inventories. Therefore, market clearing requires

$$\dot{A}_d(t) \quad = \alpha \left\{ A - A_d(t) - \textstyle\sum_{i=1}^{I} \pi_i U_i'^{-1} [\xi(t)] \right\}, \tag{13}$$

after substituting the investor's first-order condition (10). This market-clearing condition determines the inventory path given some $\xi(t)$. Aggregating (12) across all dealers, we find the condition:

$$\dot{\xi}(t) - (r + \kappa)\xi(t) \leq 0 \text{ with an equality if } A_d(t) > 0. \tag{14}$$

An equilibrium following the recovery time T_ρ is a solution $\{A_d(t), \xi(t); t \geq T_\rho\}$ to the system of differential equations (13) and (14), with the given initial condition $A_d(T_\rho)$. While we do not include $p(t)$ in the definition of an equilibrium, it can be recovered from (11), which, using the no-bubble condition, $\lim_{t\to\infty} e^{-rt} p(t)$, implies

$$p(t) = \int_t^\infty e^{-r(s-t)} \left[\xi(s) - \frac{\dot{\xi}(s)}{r + \kappa} \right] ds.$$

In a steady state, $\xi(t) = \bar{\xi}$ and $\dot{\xi}(t) = 0$ so that (14) holds with a strict inequality, and $A_d(t)$ must be equal to zero. If we set $A_d(t) = 0$ into equation (13), we find that the steady state $\bar{\xi}$ is the unique solution of

$$\sum_{i=1}^{I} \pi_i U_i'^{-1}(\bar{\xi}) = A.$$

In addition, with equation (11) we find that the steady-state price solves $r\bar{p} = \bar{\xi}$. The following proposition summarizes the equilibrium path towards the steady state.

Proposition 1 (The equilibrium path to recovery)**.** *There is a unique equilibrium path* $\{\xi(t), A_d(t) : t \geq T_\rho\}$ *and it is such that:*

(a) *For all* $t \in (T_\rho, T]$,

$$\xi(t) = \bar{\xi}e^{-(r+\kappa)(T-t)} \tag{15}$$

$$A_d(t) = e^{-\alpha(t-T_\rho)}A_d(T_\rho) + \alpha \int_{T_\rho}^{t} e^{-\alpha(t-s)} \left[A - \sum_{i=1}^{I} \pi_i U_i'^{-1}[\xi(s)]\right] ds, \tag{16}$$

where $T < \infty$ *is the unique solution to* $A_d(T) = 0$.

(b) *For all* $t \geq T$, $\{\xi(t), A_d(t)\} = (\bar{\xi}, 0)$.

According to (15), the investor's effective cost of holding the asset, $\xi(t)$, increases at rate $r + \kappa$ while dealers hold inventories; meanwhile, according to (16), the stock of assets held by dealers decreases monotonically until it is fully depleted at time T. The condition $A_d(T) = 0$ provides a relationship between the effective cost of holding the asset at the recovery time, $\xi(T_\rho) = \bar{\xi}e^{-(r+\kappa)(T-T_\rho)}$, and dealers' initial inventories, $A_d(T_\rho)$. We represent this relationship by the function $\psi(A)$ such that $\xi(T_\rho) = \psi[A_d(T_\rho)]$. Notice that $\psi'(A) < 0$, so $\xi(T_\rho)$ is decreasing in $A_d(T_\rho)$, and $\psi(0) = \bar{\xi}$. Intuitively, the larger the stock of inventories that dealers are holding at the time of the recovery, the longer it will take dealers to unwind their inventories once the recovery has occurred. But the only way dealers are willing to hold assets longer is if they make a larger capital gain, that is, if the effective cost of holding the asset at the recovery time, $\xi(T_\rho)$, is lower.

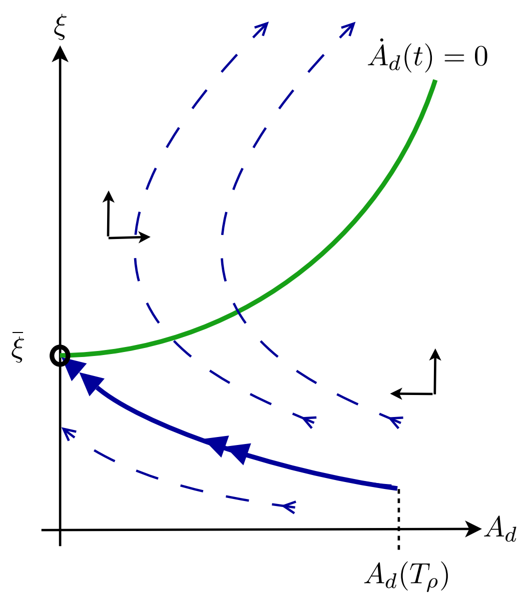

Figure 3: Phase diagram for the equilibrium recovery path

Figure 3 shows the phase diagram of the dynamic system $\{A_d(t), \xi(t)\}$ following the recovery. From (13) we see that the A_d-isocline is upward-sloping and intersects the vertical axis at the steady-state point. The sign of the derivatives $\dot{A}_d(t)$ and $\dot{\xi}(t)$ in various regions of the plane are indicated by horizontal and vertical arrows. The equilibrium trajectory of the economy is indicated in the figure by double arrows along the saddle-path, namely, $\xi(t) = \psi\left[A_d(t)\right]$. The initial condition $A_d(T_\rho)$ determines the starting point on the saddle path. The trajectories marked with dotted lines that do not follow the saddle path are solutions to the differential equations (13) and (14), but they either fail to satisfy the no-bubble condition or the requirement that the equilibrium path, $\xi(t)$, be continuous.

3.2 The crisis

In this section, we analyze the economy during the initial crisis period, $t < T_\rho$. The value functions and the asset price following the recovery, characterized in Section 3.1, are a function of time t, of the recovery time, T_ρ, and of the starting aggregate inventory of dealers. To simplify notations, we denote these functions by $V_i(a, t \,|\, T_\rho)$, $W(a_d, t \,|\, T_\rho)$, and $p(t \,|\, T_\rho)$, respectively. For all value functions and prices during the crisis, $t < T_\rho$, we add the superscript "C."

3.2.1 Dealer's and Investor's problems

At any time t, before the recovery has occurred, the dealer solves

$$\max_{q^C(s)} \mathbb{E}\left[\int_t^{T_\rho} -e^{-r(s-t)} p^C(s) q^C(s) ds + e^{-r(T_\rho - t)} W\left[a_d^C\left(T_\rho\right), T_\rho \,|\, T_\rho\right]\right], \tag{17}$$

subject to $\dot{a}_d^C(s) = q^C(s)$, $a_d^C(s) \geq 0$ for all $s \geq t$, and the initial condition $a_d^C(t) = a$. The following lemma describes the optimality conditions.

Lemma 3. *Suppose that the price path, $p^C(s)$, during the crisis is differentiable and satisfies the no-bubble condition. Then, a bounded inventory path, $a_d^C(s)$, with initial condition $a_d^C(t) = a$, solves the dealer's problem if and only if for all $s > t$:*

$$\dot{p}^C(s) + \rho\left(p(s \,|\, s) - p^C(s)\right) - r p^C(s) \leq 0, \quad \text{with equality if} \quad a_d^C(s) > 0. \tag{18}$$

From Lemma 3 we see that the flow dealers' profit during the crisis has three components: the capital gain while the economy remains in the crisis state, $\dot{p}^C(s)$, the expected capital gain,

$\rho(p(s\,|\,s) - p^C(s))$, if the economy recovers with Poisson intensity ρ, and the opportunity cost of holding the asset, $rp^C(s)$.

Following the same steps as in the previous section, it can be shown that an investor who gains access to the market at time $t < T_\rho$ with preference type i, chooses $a_i^C \geq 0$ in order to maximize

$$\mathbb{E}_i\left[\int_t^{\tilde{T}} \left(\theta + (1-\theta)\mathbb{I}_{\{s\geq T_\rho\}}\right) u_{k(s)}(a_i^C)e^{-r(s-t)}\,ds - \left(p(t) - e^{-r(\tilde{T}-t)}p(\tilde{T})\right)a_i^C\right].$$

where

$$p(\tilde{T}) = \mathbb{I}_{\{\tilde{T}<T_\rho\}}p^C(\tilde{T}) + \mathbb{I}_{\{\tilde{T}\geq T_\rho\}}p(\tilde{T}\,|\,T_\rho).$$

There are only two differences between this equation and equation (7) in the previous section: first, the period utility for the asset is scaled down by θ whenever $s \leq T_\rho$, and, second, an investor expects that the economy may have recovered by the time $\tilde{T}$, when he is able to resell the asset. The following lemma provides a simpler formulation of the investor's problem.

Lemma 4. *Let* $u_i^C(a) \equiv \frac{r+\kappa}{r+\kappa+\rho}\theta u_i(a) + \frac{\rho}{r+\kappa+\rho}U_i(a)$ *and*

$$\begin{aligned} U_i^C(a) &= \frac{(r+\kappa+\rho)\,u_i^C(t) + \delta\sum_{j=1}^I \pi_j u_j^C(a)}{r+\kappa+\rho+\delta} && (19)\\ \xi^C(t) &= (r+\kappa)\left[p^C(t) - \int_t^\infty \kappa e^{-(r+\kappa)(\tau_\kappa - t)}\left(e^{-\rho(\tau_\kappa-t)}p^C(\tau_\kappa)\right.\right.\\ &\qquad \left.\left. + \int_0^{\tau_\kappa}\rho e^{-\rho(\tau_\rho-t)}p(\tau_\kappa\,|\,\tau_\rho)\,d\tau_\rho\right)d\tau_\kappa\right]. && (20)\end{aligned}$$

Assume that $\mathbb{E}_t\left[p(s)e^{-r(s-t)}\right]$ *is decreasing in* s *and that* $p(s)$ *satisfies the no-bubble condition. Then a bounded process* $a^C(t)$ *solves the investor's problem if and only if, when the investor contacts the market with current type* i*,*

$$a^C(t) = a_i^C(t) \text{ where } U_i^{C\prime}\left[a_i^C(t)\right] = \xi^C(t). \qquad (21)$$

This is the natural counterpart of Lemma 2. As before, $\mathbb{E}_t\left[p(s)e^{-r(s-t)}\right]$ has to be decreasing in an equilibrium, otherwise the dealer's problem would not have a bounded solution. Note that the formula for $U_i^C(a)$ is similar to the one for $U_i(a)$, except that the period utility $u_i(a)$ is replaced by

$$\frac{r+\kappa}{r+\kappa+\rho}\theta u_i(a) + \frac{\rho}{r+\kappa+\rho}U_i(a).$$

Intuitively, the investors rescale his period utility by θ while keeping in mind that, before the next contact time, the recovery may arrive with Poisson intensity ρ, in which case the flow continuation utility becomes $U_i(a)$. The formula for $\xi^C(t)$ takes into account the expected capital gain that will be realized the next time the investor gains access to the market, which may be before or after the economy recovers. As before, the last two terms on the right-hand side of (20) represent the expected resale price of the asset.

3.2.2 The equilibrium path during the crisis

After differentiating condition (20), we find that

$$-rp^C(t) + \dot{p}^C(t) + \rho\left[p(t,t) - p^C(t)\right] = -\xi^C(t) + \frac{\dot{\xi}^C(t) + \rho\left[\xi(t\,|\,t) - \xi^C(t)\right]}{r+\kappa}, \tag{22}$$

where

$$\xi\left(t\,|\,T_\rho\right) = \psi\left[A_d(T_\rho)\right]e^{(r+\kappa)(t-T_\rho)}.$$

Plugging (22) back into the dealer's first-order condition (17) and aggregating, we obtain

$$\left\{\dot{\xi}^C(t) + \rho\psi[A_d^C(t)] - (r+\kappa+\rho)\,\xi^C(t)\right\}A_d^C(t) \leq 0 \text{ with an equality if } A_d^C(t) > 0. \tag{23}$$

The market clearing condition is

$$\dot{A}_d^C(t) = \alpha\left\{A - A_d^C(t) - \sum_{i=1}^{I}\pi_i U_i^{C\prime-1}[\xi^C(t)]\right\}, \tag{24}$$

which is the same as before except for the fact that $U_i'(a)$ is replaced by $U_i^{C\prime}(a)$. We can now define an equilibrium during the crisis to be a pair $\left\{\xi^C(t), A_d^C(t)\right\}$, satisfying (23) and (24).

One can easily show that the system (23) and (24) has a unique steady state $(\bar{\xi}^C, \bar{A}_d^C)$ characterized by

$$\begin{aligned}
\bar{\xi}^C &\geq \frac{\rho}{r+\kappa+\rho}\psi(\bar{A}_d^C) \text{ with an equality if } \bar{A}_d^C > 0\\
A &= \bar{A}_d^C + \sum_{i=1}^{I}\pi_i U_i^{C\prime-1}(\bar{\xi}^C).
\end{aligned}$$

Analyzing the system (23) and (24) of ODEs yields:

Proposition 2 (The equilibrium path during the crisis)**.** *Assume that $\bar{A}_d^C > 0$ and suppose $A_d^C(0) = 0$. Then, the equilibrium crisis path is unique, starts with $\xi^C(0) > \bar{\xi}^C$, and converges monotonically to the steady state, $\{\bar{\xi}^C, \bar{A}_d^C\}$.*

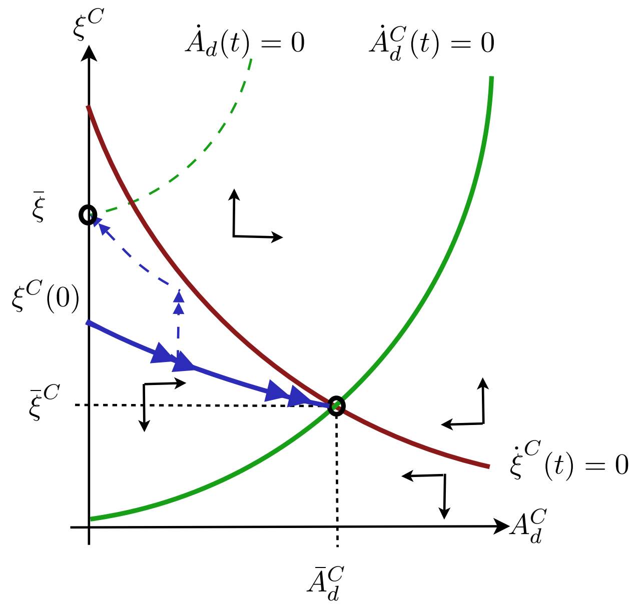

Figure 4: Phase diagram for the crisis path.

These properties can be intuitively derived using the phase diagram of Figure 4. The isocline $\dot{A}_d^C = 0$ during the crisis is represented by the upward plain curve. It is located to the right of the recovery isocline $\dot{A}_d = 0$, represented by the upward dashed curve. This is because, for any given ξ, investors' demand for the asset is lower during the crisis, and hence dealers' demand must be higher for the market to clear. The isocline $\dot{\xi}^C = 0$ is represented by the downward-sloping plain curve. Proposition 2 shows that, given the initial condition $A_d^C(0) = 0$, there is a unique saddle-path during the crisis, represented in the figure by the plain curve with double arrows, leading to the steady state, $\left(\bar{A}_d^C, \bar{\xi}^C\right)$.

3.3 Putting the crisis and the recovery together

Taken together, Propositions 1 and 2 show that the equilibrium unfolds as follows. The economy starts at $A_d^C(0) = 0$, and at the time of the crisis, $\xi^C(t)$ jumps down to the saddle-path leading to $(\bar{A}_d^C, \bar{\xi}^C)$. The economy then evolves along the crisis saddle-path until the random-recovery shock occurs. If $\bar{A}_d^C > 0$, then along the crisis saddle-path, dealers' inventories increase and $\xi^C(t)$ decreases. At the random recovery time, the system jumps to the recovery saddle-path leading to $\left(0, \bar{\xi}\right)$. This is the saddle path of Proposition 1, indicated in Figure 3 by the dashed curve with double arrows. At the time the recovery shock occurs, the cost $\xi(t)$ of holding the asset jumps up, and dealers begin selling their inventories gradually until they are completely depleted. We summarize these findings in the following corollaries:

Corollary 1 (Crisis and recovery dynamics when dealers provide liquidity)**.** *Suppose* $\bar{A}_d^C > 0$. *At the time of the crisis,* $t = 0$*, the price* $p(t)$ *jumps down. Then, as the crisis unfolds, for* $t \in (0, T_\rho)$*, dealers' inventories increase towards* $\bar{A}_d^C$ *while the price continues to decrease. At the time of the recovery,* $t = T_\rho$*, the price jumps up. During the recovery,* $t \in [T_\rho, \infty)$*, dealers' inventories decrease towards zero, and the price continues to increase towards* $\bar{\xi}/r$*.*

The corollary is illustrated in the left panel of Figure 5. While the analysis in the first paragraph of this section showed that the effective cost $\xi^C(t)$ is decreasing during the crisis, Corollary 1 shows that this is also true for the price, $p^C(t)$. Note that our profit maximizing and atomistic dealers find it optimal to buy in a down market (see Ross and Sofianos, 1998, for evidence of such behavior). They do not prefer to wait and buy at a lower price, since by waiting they may "miss" the capital gain at the recovery time, T_ρ.

There is a simple intuition for why the price has to fall during the crisis, even without the arrival of further bad news. Dealers anticipate that, as they accumulate inventories, they will take longer to unwind their asset positions. Thus, they have to be compensated by a larger capital gain, implying that the price has to fall by more before the recovery time. Therefore, search frictions create a price divergence similar to that of Kondor (2009), without putting any limit on dealers' capital.

As will become clear in the next section, for some parameters $\bar{A}_d^C = 0$, meaning that dealers do not accumulate any inventories during the crisis. In such cases, we obtain the dynamics illustrated in the right-panel of Figure 5.

Corollary 2 (Crisis and recovery dynamics when dealers do not provide liquidity)**.** *Suppose* $\bar{A}_d^C = 0$*. Then, dealers do not hold inventories during the crisis. At the time of the crisis,* $t = 0$*, the price,* $p(t)$*, jumps down and remains constant during the crisis. At the recovery time,* $t = T_\rho$*, the price jumps up to its steady-state level,* $\bar{\xi}/r$*.*

4 Implications

We first study how dealers' incentives to provide liquidity are influenced by the two key OTC market frictions: i) the fact that locating counterparties for trade is time-consuming, ii) and prices are determined through bargaining. These frictions are captured by α and η. In some instances, e.g., the frictionless case $\alpha = \infty$, it is clear that dealers' liquidity provision does not improve welfare and dealers do not provide liquidity in equilibrium. However, our model

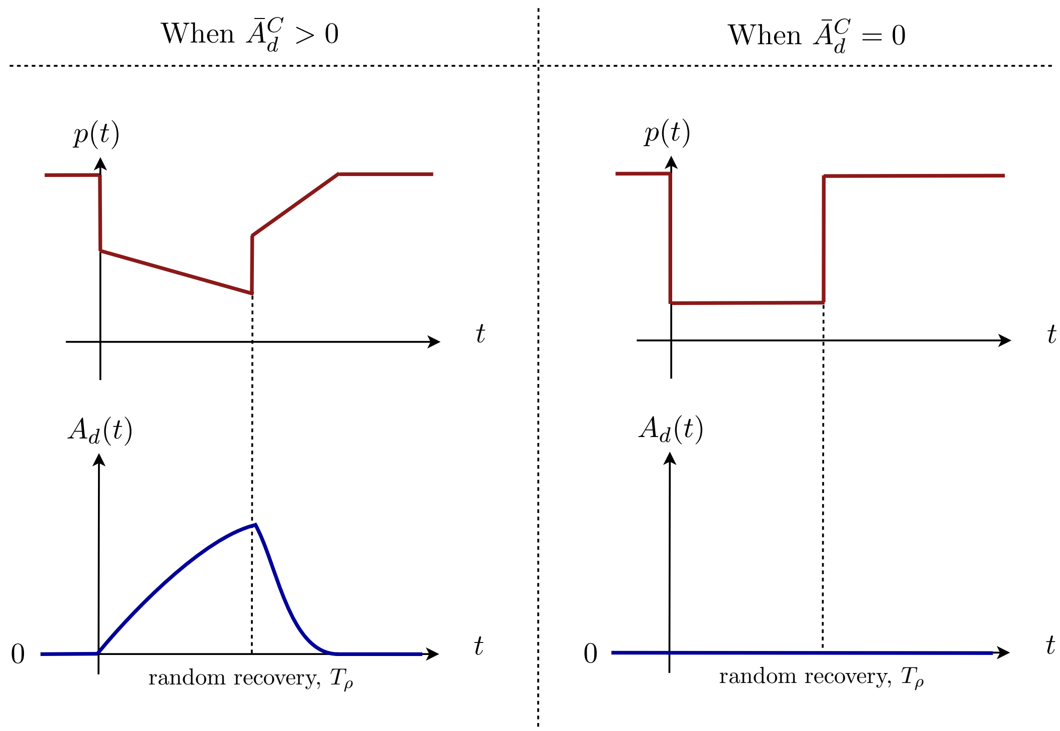

Figure 5: The price and inventory paths when dealers provide liquidity (left panel) and when they don't (right panel).

shows that, in other cases, dealers may fail to provide liquidity when it would be socially optimal for them to do so. In Section 4.3, we discuss various policy interventions that may be put in place to mitigate a socially inefficient failure of dealers to provide liquidity to investors.

4.1 Dealers' incentives to provide liquidity

We will assume from now on that investors have an isoelastic utility function $u(a) = a^{1-\sigma}/(1-\sigma)$ with $\sigma > 0$, and that the idiosyncratic preference shock is multiplicative, i.e., $u_i(a) = \varepsilon_i u(a)$ with $\varepsilon_i \in \{\varepsilon_1, ..., \varepsilon_I\}$.[8] In that case, we know from Lemma 2 that, after the recovery, investors' equivalent utility flow is $U_i(a) = \bar{\varepsilon}_i a^{1-\sigma}/(1-\sigma)$, where

$$\bar{\varepsilon}_i = \frac{(r+\kappa)\varepsilon_i + \delta \sum_{j=1}^{I} \varepsilon_j}{r+\kappa+\delta}. \tag{25}$$

[8]In Lagos, Rocheteau, and Weill (2007) we generalize some results (e.g., the condition under which dealers accumulate asset inventories) for arbitrary utility functions.

Similarly, Lemma 4 shows that, during the crisis, $U_i^C(a) = \bar{\varepsilon}_i^C a^{1-\sigma}/(1-\sigma)$, where:

$$\bar{\varepsilon}_i^C = \frac{(r+\kappa+\rho)\varepsilon_i^C + \delta \sum_{j=1}^I \pi_j \varepsilon_j^C}{r+\kappa+\rho+\delta} \quad \text{and} \quad \varepsilon_i^C = \frac{(r+\kappa)\theta\varepsilon_i + \rho\bar{\varepsilon}_i}{r+\kappa+\rho}. \tag{26}$$

This functional form allows us to derive a simple condition on exogenous parameters under which dealers provide no liquidity.

4.1.1 A simple condition for no liquidity provision

From Lemma 3, dealers find it strictly optimal to provide no liquidity if and only if:

$$\frac{\rho\left[p(t\,|\,t) - p^C(t)\right] + \dot{p}^C(t)}{p^C(t)} < r \Leftrightarrow \frac{\rho\left[\xi(t\,|\,t) - \xi^C(t)\right] + \dot{\xi}^C(t)}{\xi^C(t)} < r+\kappa. \tag{27}$$

In words, the condition specifies that the expected return of purchasing the asset at time t and re-selling it at time $t+dt$ (the left side of (27)) must be less than the rate at which dealers can borrow funds (the right side of (27)).

Note that, if (27) holds, then dealers' aggregate inventory position is equal to zero during the crisis, $A_d^C(t) = \dot{A}_d^C(t) = 0$. Together with (24), this implies that $\xi^C(t)$ is constant and equal to $\bar{\xi}_0^C$, solving

$$\sum_{i=1}^I \pi_i U_i^{C\prime -1}\left(\bar{\xi}_0^C\right) = A \quad \Longleftrightarrow \quad \bar{\xi}_0^C = A^{-\sigma}\left(\sum_{i=1}^I \pi_i \left(\bar{\varepsilon}_i^C\right)^{\frac{1}{\sigma}}\right)^{\sigma}, \tag{28}$$

given our functional form $U_i^C(a) = \bar{\varepsilon}_i^C a^{1-\sigma}/(1-\sigma)$. At time of the random-recovery shock, the economy jumps to its long-run steady state, which under our functional form is:

$$\bar{\xi} = A^{-\sigma}\left(\sum_{i=1}^I \pi_i \left(\bar{\varepsilon}_i\right)^{\frac{1}{\sigma}}\right)^{\sigma}. \tag{29}$$

Taken together, (27), (28) and (29) imply

Proposition 3. *Assume $u_i(a) = \varepsilon_i a^{1-\sigma}/(1-\sigma)$. Then, dealers find it strictly optimal to provide no liquidity during the crisis if and only if*

$$\frac{\sum_{i=1}^I \pi_i \left(\bar{\varepsilon}_i^C\right)^{\frac{1}{\sigma}}}{\sum_{i=1}^I \pi_i \left(\bar{\varepsilon}_i\right)^{\frac{1}{\sigma}}} > \left(\frac{\rho}{r+\kappa+\rho}\right)^{\frac{1}{\sigma}}. \tag{30}$$

Sufficient conditions for (30) to hold are: i) α approaches infinity; ii) $r+\kappa = r+\alpha(1-\eta)$ approaches zero.

Condition (30), under which dealers choose not to provide liquidity, depends on investors' preferences, the characteristics of the crisis, and the structure of the market. Focusing on the key OTC trading frictions, the main insight of Proposition 3 is that dealers' incentives to accumulate asset inventories vary in a non-monotonic fashion with the extent of the trading frictions: if $\kappa = \alpha(1-\eta)$ is very large or very low, then dealers do not intervene to mitigate the selling pressures.

To see why this should be so, consider first the case where α goes to infinity (which implies that κ goes to infinity as long as $\eta < 1$) and the economy approaches the frictionless Walrasian benchmark. In this case, both dealers and investors are able to trade the asset continuously over time, but dealers get no direct (marginal) utility from holding the asset, while investors do. As a consequence, there are no private or social gains from having dealers hold asset inventories. It is not surprising, then, that dealers provide no liquidity when trading frictions are small.

Consider next the case where α approaches zero, implying that it takes a long time for investors to locate counterparties. Dealers, in contrast, can trade continuously. One might conjecture that this market timing advantage over investors would give dealers a strong incentive to provide liquidity: they could accumulate assets during the market crash and resell them very quickly to the most eager asset holders when the economy recovers. This would allow them to reap the intertemporal gains from trade implied by variations in asset demands before and after the crisis. Our analysis reveals that this intuition is misleading because the gains from these intertemporal trades vanish when trading frictions are large. Indeed, when α is very low, investors who have the opportunity to readjust their asset holdings anticipate that they will be holding their assets for a long period of time (since the average holding period of the asset is $1/\alpha$). As a consequence, investors choose asset positions based on their *average marginal utility* for the asset instead of their current marginal utility. Formally, investors' effective preference shocks, $\bar{\varepsilon}_i^C$ and $\bar{\varepsilon}_i$, both converge towards $\bar{\varepsilon}$ as $r + \kappa$ goes to zero. Since all investors enjoy approximately the same expected marginal utility from holding the asset between two consecutive contacts with dealers, they find it optimal to hold approximately the same position. Clearly, this implies that there are very small gains to be had from reallocating the asset between two investors before and after the crisis, and dealers cannot reap many benefits from their ability to reallocate the asset faster than investors over time. Put differently, in markets with very severe trading frictions, investors do not demand much liquidity from dealers, thereby reducing dealers' incentive to provide liquidity.

4.1.2 A numerical example

So far, we have shown that if we consider a spectrum of asset markets going from very liquid to very illiquid markets, dealers do not provide liquidity at either end of the spectrum. We now show, by way of a numerical example, that there are parameterizations for which condition (30) is not satisfied for intermediate values of α. That is, in OTC markets where trading frictions are neither too mild nor too severe, dealers find it optimal to provide liquidity. The green shaded regions in Figure 6 represent parameter values for which dealers find it optimal to provide liquidity in times of crisis. In each panel, we let the two parameters in the axes vary and keep the rest fixed at some benchmark values. All panels have the extent of the search friction, α, on the horizontal axis.

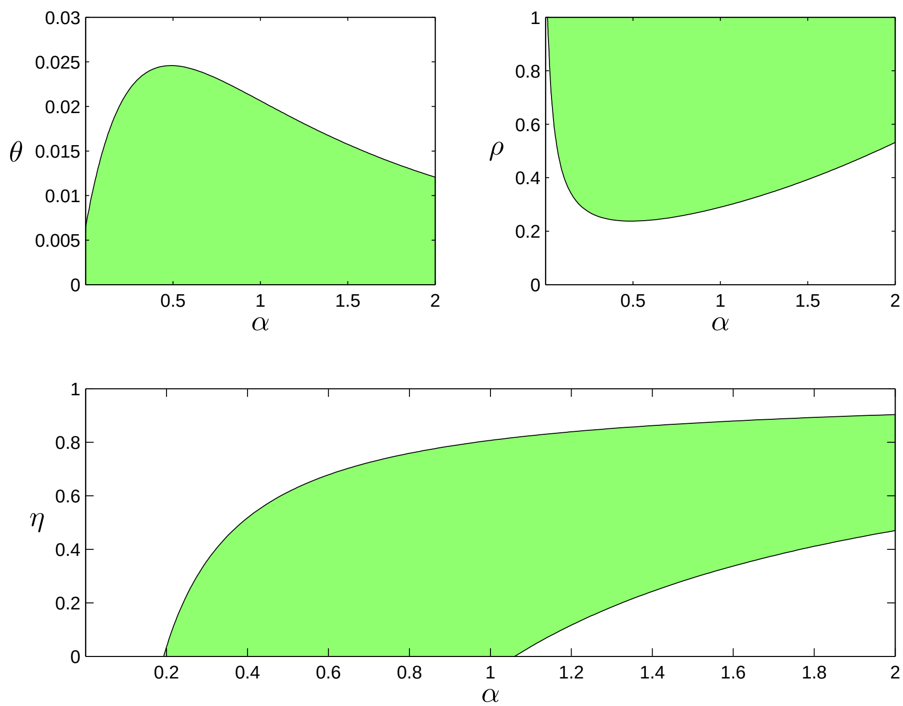

Figure 6: Parameterizations for which dealers provide liquidity. The benchmark parametrization is: $\sigma = 0.5$, $r = 0.05$, $\pi_1 = \pi_2 = 0.5$, $\alpha = 0.5$, $\delta = 1$, $\rho = 0.3$, $\theta = 0.02$, $\eta = 0$, and $A = 1$.

The first and second panels of Figure 6 relate liquidity provision to the characteristics of the crisis, θ and ρ. They show that dealers are more likely to accumulate asset inventories when the market crash is severe (θ is low) and expected to be short-lived (ρ is large). Intuitively, if the crash is very sharp, then there are large gains from reallocating the asset from investors during the crisis to the investors once the recovery has occurred. Moreover, if the crisis is expected to be short-lived, then the opportunity cost from having dealers holding the asset instead of investors is small.

In order to interpret the lower panel of Figure 6, it is useful to remember that the condition in Proposition 3 depends on $\kappa = \alpha(1-\eta)$, the effective degree of frictions in the economy, but not on α and η individually. So an increase in dealers' bargaining power produces the same effects in terms of liquidity provision as a reduction in α. In particular, if α is very large, then κ varies from 0 to a large number as η varies from 1 to 0. From Proposition 3, if agents are sufficiently patient, it then follows that dealers will not provide liquidity if either dealers have very little argaining power (η is close to zero) or if they have a great deal of bargaining power (η is close to one).

4.2 Liquidity provision and welfare

Next, we turn to the normative implications of the model. We seek to identify circumstances under which dealers do not provide liquidity even though, from a benevolent planner's viewpoint, there would be social gains from having them hold inventories.

With no loss of generality we can measure social welfare as the sum of the utilities of investors and dealers.[9] Also, we can omit the utility of investors before their first contact with the market. The welfare criterion takes then the simple form

$$\mathcal{W}^C = \mathbb{E}\left[\int_0^{T_\rho}\sum_{i=1}^{I}\alpha\pi_i\frac{\hat{\bar{\varepsilon}}_i^C u(a_i^C(t))}{r+\alpha}e^{-rt}\,dt + \int_{T_\rho}^{\infty}\sum_{i=1}^{I}\alpha\pi_i\frac{\hat{\bar{\varepsilon}}_i u(a_i(t))}{r+\alpha}e^{-rt}\,dt\right],$$

where T_ρ is the random time at which the economy recovers, α is the flow of investors in contact with dealers at time t, and π_i is the fraction of investors of type i among all these investors. The preference shocks $\hat{\bar{\varepsilon}}_i$ and $\hat{\bar{\varepsilon}}_i^C$ are obtained by setting $\eta = 0$ in the expressions (25) and (26) for $\bar{\varepsilon}_i$ and $\bar{\varepsilon}_i^C$.

In the planner's objective, the utilities $\hat{\bar{\varepsilon}}_i^C u(a_i^C(t))/(r+\alpha)$ and $\hat{\bar{\varepsilon}}_i u(a_i(t))/(r+\alpha)$ represent the "true" expected discounted utilities of an investor at time t until his next contact with the market, before and after the recovery, respectively. Crucially, these differ from the utilities that an individual investor uses to calculate his optimal asset holding. Indeed, an investor anticipates that he always loses a fraction of the gains from trading with dealers. The planner, on the other hand, takes into account that the gains from trade lost by the investors are, in

[9]Because agents have quasilinear utility, maximizing this criterion subject to search frictions will characterize *all* constrained-optimal Pareto asset allocations, i.e., all feasible asset allocations that cannot be Pareto improved by choosing another feasible allocation and making time-zero transfers of the numéraire good.

fact, enjoyed by dealers.

The planner maximizes the above objective by choosing the asset holdings $\{a_i(t)\}_{i=1}^I$ of those investors contacting the market at time t and the asset holdings $A_d(t)$ of dealers. The allocation chosen by the planner must be, of course, feasible given the search frictions. One easily shows that this constraint leads to the ODE:

$$\dot{A}_d(t) = \alpha \left\{ A - A_d(t) - \sum_{j=1}^{I} \pi_j a_j(t) \right\},$$

which is, unsurprisingly, the same as the ODE governing market clearing in the equilibrium. Analyzing the planner's control problem we obtain the following result.

Proposition 4. *The equilibrium is socially efficient if and only if $\eta = 0$. It is strictly socially optimal that dealers provide no liquidity if and only if*

$$\frac{\sum_{i=1}^{I} \pi_i \left(\hat{\hat{\varepsilon}}_i^C\right)^{\frac{1}{\sigma}}}{\sum_{i=1}^{I} \pi_i \left(\hat{\hat{\varepsilon}}_i\right)^{\frac{1}{\sigma}}} \geq \left(\frac{\rho}{r+\kappa+\rho}\right)^{\frac{1}{\sigma}}. \tag{31}$$

The proposition shows that, in markets where dealers have the ability to extract some rent from their trades with investors, the choice of asset holdings is distorted: investors choose asset positions that reduce the transaction fees they will have to pay in the future when they will have to readjust their asset holdings.

Next, we provide a condition where the lack of liquidity provision is socially inefficient for limiting economies in which agents are infinitely patient ($r \to 0$).

Corollary 5. *Consider an economy such that (31) does not hold as $r \to 0$. Then there is some $\bar{\eta} < 1$ such that for all $\eta > \bar{\eta}$, dealers do not hold any inventories even though the planner's allocation would require them to do so.*

The corollary starts from an economy with $r = 0$, where it is socially optimal to have dealers provide liquidity. Then, as η approaches one, the effective trading rate, κ, approaches zero. It then immediately follows from Proposition 3 that, if η is sufficiently high, then in equilibrium dealers won't provide any liquidity. While Proposition 5 assumes $r \to 0$, the lower panel of Figure 6 shows numerical examples of economies with $r > 0$ where dealers underprovide liquidity.

4.3 Policy implications

What are the policy responses to insufficient liquidity provision?

4.3.1 Capital injections

By assumption, our dealers have "deep pockets," i.e., they have enough capital to buy any quantity of assets. It follows from this assumption that supplying more capital to dealers will prove ineffective: in the model, if a policy-maker makes a lump-sum transfer of the numéraire good to dealers, they will hoard it instead of using it to purchase assets from outside investors. Such a capital injection is ineffective because, as revealed by Corollary 5, the root of the problem is not a credit market imperfection, but the very structure of OTC markets: the fact that dealers are able to extract some rents from their trades with investors.

This is not to say that credit market imperfections do not matter. We have seen in Figure 6 that there are parameter values for which dealers will provide liquidity to investors (the green region). If dealers face a capital constraint, and if η is close to 0, then capital injections would be welfare improving for such parameter values. Our main insight is that this policy response is ineffective if the lack of liquidity provision comes from OTC market frictions rather than from capital constraints.

4.3.2 Market structure

If the government is able to reform the structure of the OTC markets, our model suggests that it should encourage market reforms that facilitate trades between investors and dealers (increase α) and that erode dealers' market power (reduce η). The view that eroding market power is socially beneficial lends support to the Euronext rule that Designated Market Markers should commit to a minimum spread (see Menkveld and Wang, 2009).

In practice, the trading frictions can be reduced in several ways. The regulator can promote standardization of the assets traded in OTC markets, disclosure of information regarding the assets' characteristics, and the development of electronic trading platforms that facilitate and speed up trades. The regulator can also maintain market liquidity by offering safer and more recognizable collateral, which may reduce counterparty risk,[10] and by preventing the failure of large dealers-brokers.

[10] The adverse selection problems induced by unobserved counterparty risk may worsen search frictions: for instance, Hopenhayn and Werner (1996) show how information problems in bilateral meetings may reduce the probability of a trade.

4.3.3 Government asset purchases in OTC markets

OTC markets may be organized in a decentralized fashion for efficiency reasons. For instance, in credit derivative markets, the search friction may be the unavoidable consequence of investors' desire to enter into finely customized contracts. Similarly, the bargaining power of dealers may be required to cover *ex-ante* entry costs in the market.[11]

In what follows we propose a policy exercise that does not require taking a stand on these questions. We take the point of view of a government at the beginning of a severe crisis, which takes the OTC market structure as given. Since capital injections are ineffective, we ask whether there is room for the government to step into the interdealer market and accumulate assets on its own account, effectively acting as a "liquidity provider of last resort." Note that, even though Proposition 4 showed that the socially optimal allocation prescribes that dealers hold inventories, it is a priori not obvious that the provision of liquidity by the government would be welfare-improving. Indeed, while the planner is only constrained by the search frictions, as measured by α, the government is also constrained by η, i.e., it takes as given the fact that, in an equilibrium, investors' asset positions will remain distorted by the positive bargaining power of dealers.

We carry out the policy experiment in an economy where dealers find it strictly optimal to provide no liquidity. We ask whether a benevolent government would find it worthwhile to conduct the following small asset purchase in the interdealer market. During the crisis, $t \in [0, T_\rho)$, the government purchases asset inventories $A_g^C(t) = \omega A \left(1 - e^{-\alpha t}\right)$ for some small ω. At the time of the recovery, the government sells its assets so that the market price grows at rate r. The purchases are financed via lump-sum taxes. As will become clear, we can pick ω small enough so that, in an equilibrium where the government follows this trading strategy, dealers still find it optimal to provide no liquidity.

Notice first that, during the crisis:

$$\alpha A_g^C(t) + \dot{A}_g^C(t) = \alpha \omega A$$

remains constant. It follows from the market-clearing condition (24) that during the crisis,

[11] See, e.g., Duffie, Gârleanu, and Pedersen (2005) for a model where dealers choose their search intensity, Lagos and Rocheteau (2009) for a model where the contact rate between dealers and investors is endogeneized by free entry of dealers, and Mínguez Afonso (2008) for where it is endogenized by the free entry of investors. It is difficult to study dealers' liquidity provision in these models, because it requires analyzing their out-of-steady-state dynamics. Out of the steady state, entry depends on the entire distribution of investors' states (preference types and asset holdings), which determines their expected profits, and investors' asset holdings depend on the expected entry of dealers over time.

the effective cost of holding the asset, ξ^C, that clears the market is also constant and solves

$$\sum_{i=1}^{I} \pi_i \left(\frac{\xi^C}{\bar{\varepsilon}_i^C}\right)^{-\frac{1}{\sigma}} = A(1-\omega) \quad \Longleftrightarrow \quad \xi^C = (1-\omega)^{-\sigma}\bar{\xi}_0^C,$$

where $\bar{\xi}_0^C$ is the effective cost of holding the asset in the absence of government intervention. Consider next what happens at the onset of the recovery. The government sells its asset at a speed that guarantees that the price grows at the discount rate, r. Or, equivalently:

$$\xi(t \,|\, T_\rho) = e^{-(r+\kappa)(T-t)}\bar{\xi},$$

where T is the time at which the government's asset inventories are depleted. The corresponding government inventories, $A_g(t \,|\, T_\rho)$, follows directly after plugging $\xi(t \,|\, T_\rho)$ into the market clearing condition, holding dealers' inventories equal to zero. Note that an equivalent strategy would be for the government to sell all its inventories to dealers at time T_ρ, and let dealers trade afterwards. Our main result is:

Proposition 6 (Welfare impact of a direct asset purchase)**.** *Consider an economy such that condition (30) holds, and let $\Delta\mathcal{W}(\omega)$ denote the change in social welfare induced by a direct asset purchase of size ω. Then,*

$$\lim_{\omega\to 0} \frac{\Delta\mathcal{W}(\omega)}{\omega} = -(r+\alpha)\mu^C + \rho(\mu - \mu^C), \tag{32}$$

where $\mu^C = \sum_i \pi_i \hat{\bar{\varepsilon}}_i^C u'(a_i^C)a_i^C$, and $\mu = \sum_i \pi_i \hat{\bar{\varepsilon}}_i u'(a_i)a_i$, while a_i^C and a_i are the investor's asset holdings without government intervention, during the crisis and during the recovery.

The proposition shows that, in order to evaluate the welfare impact of its outright purchase, the government uses the "true" undistorted preference shocks, $\hat{\bar{\varepsilon}}_i$ and $\hat{\bar{\varepsilon}}_i^C$, instead of the distorted ones, $\bar{\varepsilon}_i$ and $\bar{\varepsilon}_i^C$, used by investors. The proposition also reveals the manner in which the government is constrained by the market structure: welfare is evaluated at the margins implied by the distorted asset holdings, a_i^C and a_i, chosen by investors when they bargain with dealers.

The two terms in (32) show the tradeoff faced by the government. The first term captures the foregone utility from having the government hold assets instead of investors. The second term represents the welfare gain from reallocating the assets from investors during the crash to investors during the recovery. Numerical calculations, shown in Figure 7, suggest that, for some parameters, we can have:

$$
\begin{aligned}
-(r+\kappa)\bar{\xi}_0^C + \rho\left(\bar{\xi} - \bar{\xi}_0^C\right) &< 0, \\
-(r+\alpha)\mu^C + \rho\left(\mu - \mu^C\right) &> 0.
\end{aligned}
$$

That is, it can be the case that the government finds it optimal to hold inventories, notwithstanding investors' distorted asset positions, and at the same time dealers do not − which implies that the government makes negative expected profit from this trade. So our model predicts that direct purchases of assets by the government can raise society's welfare when there is insufficient liquidity provision. Moreover, in our model, these government purchases do not substitute for capital injections to dealers. Direct purchases are socially beneficial because of the trading frictions distorting dealers' incentives to provide liquidity in OTC markets.

This policy experiment shows that, in some circumstances, there exists a welfare-improving policy intervention with the following features: it is small, it does not stimulate dealers' inventory accumulation, and the government makes negative expected profit. But one should bear in mind that an *optimal* policy intervention would not necessarily share these features. Characterizing the optimal policy would require setting up and solving the Ramsey problem, a question that we leave open for future work.

5 Conclusion

We have developed a model where several of the policy discussions surrounding the recent financial crisis can be analyzed. For instance, our model suggests circumstances in which the provision of funds to OTC market dealers might prove ineffective. This could help explain why most of the auctions from the Term Securities Lending Facility–a facility introduced by the Federal Reserve to allow primary dealers to borrow Treasury securities against less liquid collateral–were undersubscribed (e.g. Cecchetti, 2008). Our model also shows that direct purchases of assets by the government can be welfare improving. This may help justify the Federal Reserve decision to announce, during March 2009, that it would purchase up to $1.25 trillion of mortgage-backed securities. Lastly, our policy implications for reforming market structure are reminiscent of the framework for financial stability that the Group of Thirty published in 2009, which recommends "improvements to the infrastructure supporting the OTC derivatives markets" and an enhanced "disclosure and dissemination regime for

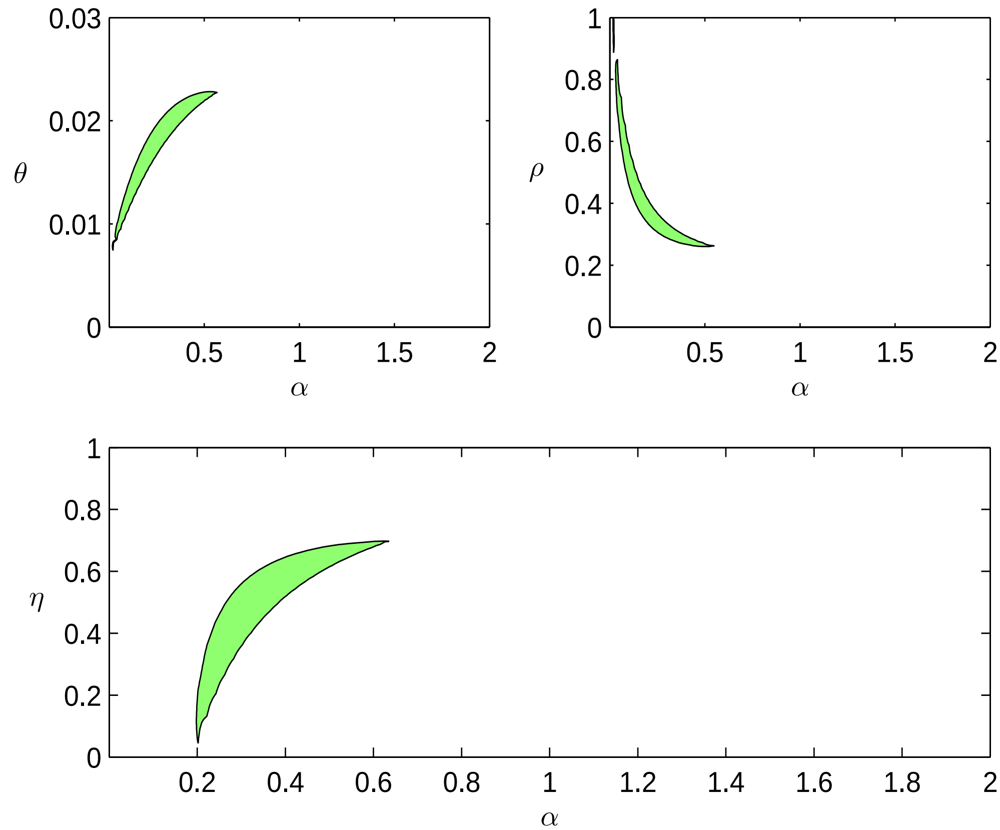

Figure 7: Parameterizations for which the government find it strictly optimal to provide liquidity but dealers don't. The benchmark parametrization is: $\sigma = 0.5$, $r = 0.05$, $\pi_1 = \pi_2 = 0.5$, $\alpha = 0.5$, $\delta = 1$, $\rho = 0.3$, $\theta = 0.02$, $\eta = 0.5$, and $A = 1$.

asset-backed and other structured fixed income financial products." They are also consistent with the recommendations of the chairman of the Federal Reserve to strengthen the financial infrastructure (e.g., Bernanke, 2009b).

A Proofs

A.1 Proof of Lemma 1

First, we substitute the constraint $\dot{a}_d(s) = q(s)$ into the dealers objective and integrate by part:

$$\int_t^T e^{-r(s-t)} p(s)\dot{a}_d(s)\,ds = a_d(t)p(t) + \int_t^T e^{-r(s-t)}\left[\dot{p}(s) - rp(s)\right] a_d(s)\,ds - a_d(T)p(T)e^{-r(T-t)}.$$

Keeping in mind that $a_d(t) = a$, letting $T \to \infty$ and using the no-bubble condition, we find that the value of the inventory path $a_d(s)$ is:

$$p(t)a(t) + \int_t^\infty e^{-r(s-t)}\left[\dot{p}(s) - rp(s)\right] a_d(s)\,ds.$$

Clearly, the condition of the Lemma is sufficient. For necessity, note that if there was some time $s > 0$ such that $\dot{p}(s) - rp(s) > 0$, then a dealer could improve her utility by accumulating more inventory around s, and the dealer's problem would not have any bounded solution.

A.2 Proof of Lemma 2 and Lemma 4

A.2.1 Preliminary Results

We let the random flow utility of an investor at time t be $u(a,t)$, where we use the time argument "t " as a short-hand for the investor's current idiosyncratic and aggregate preference shock. To simplify notations, we measure time from the point of a given contact with a dealer. We let $0 = T_0 < T_1 < T_2 < \ldots$ be the sequence of the investor's contact times with dealers, N_t be the number of contact times during $[0,t]$, and θ_t be the last contact time before t. Then, for any asset plan, a, we calculate the inter-temporal utility over $[0,t]$:

$$V_0^t(a) \equiv \int_0^t u[a(s),s]e^{-rs}\,ds - \sum_{n=1}^{N_t} p(T_n)e^{-rT_n}\left[a(T_n) - a(T_{n-1})\right],$$

along a realization of the contact time and type processes. This utility can be decomposed as

$$V_0^t = U_0^t - B_0^t + p(T_1)e^{-rT_1}a(0) - p(\theta_t)a(\theta_t)e^{-r\theta_t},$$

where

$$\begin{aligned}
U_0^t(a) &= \int_0^t u\left[a(s),s\right]e^{-rs}\,ds,\\
B_0^t(a) &= \sum_{n=1}^{N_t-1} a(T_n)\left[p(T_n)e^{-rT_n} - p(T_{n+1})e^{-rT_{n+1}}\right].
\end{aligned}$$

We consider portfolio plans a that are bounded, and such that the intertemporal utility $\mathbb{E}\left[V_0^\infty(a)\right]$ is well defined. We first establish:

Lemma 5. *As $t \to \infty$, $\mathbb{E}\left[U_0^t(a)\right]$ and $\mathbb{E}\left[B_0^t(a)\right]$ converge to finite limits, and $\mathbb{E}\left[p(\theta_t)e^{-r\theta_t}a(\theta_t)\right]$ converges to zero.*

Because of the no-bubble condition $\lim_{t\to\infty} p(t)e^{-rt} = 0$ and the fact that $a(t)$ is bounded, we have that $\lim_{t\to\infty} p(t)a(t)e^{-rt} = 0$. Since θ_t goes to infinity almost surely, it follows that $\lim_{t\to} \mathbb{E}\left[p(\theta_t)e^{-r\theta_t}a(\theta_t)\right] = 0$ as well.

Let's turn to with $\mathbb{E}\left[U_0^t(a)\right]$. When the investor's utility is bounded below, then the result follows from the assumption that the portfolio plan, a, is bounded. When the investor's utility is unbounded below and bounded above, we can assume without loss of generality that it is negative. Then $\mathbb{E}[U_0^t]$ is decreasing and thus converges either to some finite or some infinite limit. The limit, in turn, must be finite because

$$\mathbb{E}[U_0^t] = \mathbb{E}[V_0^t] + \mathbb{E}[B_0^t] - \mathbb{E}[p(T_1)e^{-rT_1}a(0)] \geq \mathbb{E}[V_0^t] - \mathbb{E}[p(T_1)e^{-rT_1}a(0)],$$

where the inequality follows because $p(t)e^{-rt}$ is decreasing and B_0^t is therefore positive. Because $\mathbb{E}[V_0^\infty]$ is well defined, the right-hand side of the inequality is bounded below, implying that $\mathbb{E}[U_0^t]$ has a finite limit. It then immediately follows that

$$\mathbb{E}\left[B_0^t\right] = -\mathbb{E}\left[V_0^t\right] + \mathbb{E}\left[U_0^t\right] + \mathbb{E}[p(T_1)e^{-rT_1}a(0)] - p(\theta_t)e^{-r\theta_t}a(\theta_t).$$

also converges to some finite limit.

Lemma 6. *An investor's intertemporal utility is*

$$\mathbb{E}\left[V_0^\infty\right] = (r+\kappa)^{-1}\mathbb{E}\left[\sum_{n=1}^{\infty} e^{-rT_n}\left\{U\left[a(T_n),T_n\right] - \xi(T_n)a(T_n)\right\}\right]. \tag{33}$$

where

$$\begin{aligned} U\left[a(T_n),T_n\right] &= (r+\kappa)\mathbb{E}\left[\int_{T_n}^{T_{n+1}} u\left[a(s),s\right]e^{-r(s-T_n)}\,ds \,\middle|\, T_n\right] \\ \xi(T_n) &= p(T_n) - \mathbb{E}\left[p(T_{n+1})e^{-r(T_{n+1}-T_n)}\,\middle|\,T_n\right]. \end{aligned}$$

To show that result, write

$$\begin{aligned} \mathbb{E}\left[B_0^\infty\right] &= \mathbb{E}\left[\sum_{n=1}^{\infty} a(T_n)\mathbb{E}\left[p(T_n)e^{-rT_n} - p(T_{n+1})e^{-rT_{n+1}}\,\middle|\, T_n\right]\right] \\ &= (r+\kappa)^{-1}\mathbb{E}\left[\sum_{n=1}^{\infty} a(T_n)\xi(T_n)e^{-rT_n}\right], \end{aligned}$$

by definition of $\xi(T_n)$. In addition note that, when u is bounded below, we can without loss of generality assume that it is positive, and we have

$$u[a(s),s]e^{-rs}\mathbb{I}_{\{s\leq\theta_t\}} \leq u[a(s),s]e^{-rs}\mathbb{I}_{\{s\leq t\}} \leq u[a(s),s],$$

and $u[a(s),s]\mathbb{I}_{\{s\leq\theta_t\}} \nearrow u[a(s),s]$ as t goes to infinity. The same reasoning go through with opposite inequalities when u is negative. Therefore, an application of the dominated convergence theorem implies that

$$\begin{aligned} \mathbb{E}\left[U_0^\infty\right] &= \lim_{t\to\infty}\mathbb{E}\left[\int_0^{\theta_t} u(a(s),s)e^{-rs}\,ds\right] = \lim_{t\to\infty}\mathbb{E}\left[\sum_{n=1}^{N_t-1}\int_{T_n}^{T_{n+1}} u(a(s),s)e^{-rs}\,ds\right] \\ &= (r+\kappa)^{-1}\mathbb{E}\left[\sum_{n=1}^{\infty} e^{-rT_n}U(a(T_n),T_n)\right], \end{aligned}$$

where the last equality follows by taking expectations of each term in the sum with respect to T_n.

A.2.2 Necessary and sufficient condition

For the "only if" part of the two Lemma, it is clear from (33) that an optimal portfolio strategy should maximize each term $U\left[a(T_n),T_n\right] - \xi(T_n)a(T_n)$, implying the investor's first-order condition. For the "if" part, we consider a plan a that satisfies the first-order conditions and compare it to some other plan a'. We find

$$\begin{aligned} &\mathbb{E}[V_0^\infty(a) - V_0^\infty(a')] \\ = \;& \mathbb{E}\left[\sum_{n=1}^{\infty} e^{-rT_n}\left(U(a(T_n),T_n) - U(a'(T_n),T_n) - \xi(T_n)\left(a(T_n)-a'(T_n)\right)\right)\right] \\ \geq \;& \mathbb{E}\left[\sum_{n=1}^{\infty} e^{-rT_n}\Big(U_a(a(T_n),T_n) - \xi(T_n)\Big)\Big(a(T_n)-a'(T_n)\Big)\right] \geq 0, \end{aligned}$$

where the first inequality follows because of concavity, and the second inequality follows because of the first-order condition in the two Lemma.

A.2.3 The expression for $U_i(a)$ and $\xi(t)$ along the recovery path

The flow inter-contact time utility is $(r+\kappa)^{-1}U\left[a(T_n),T_n\right]=(r+\kappa)^{-1}U_{i(T_n)}\left[a(T_n)\right]$, where $U_i(a)$ is defined in equation (8) of the lemma. To see why, let

$$\tilde{V}_i(a,t)=\mathbb{E}_i\left[\int_0^{\tilde{T}} e^{-rs}u_{k(t+s)}(a')\,ds\middle|k(t)=i\right].$$

By the Markovian nature of the process $k(t)$, $\tilde{V}_i(a,t)$ only depends on t through the condition $k(t)=i$ which is already captured by the subscript i. Therefore, hereafter we will slightly abuse notation and write $\tilde{V}_i\left(a\right)$ for $\tilde{V}_i\left(a,t\right)$. Denote $\hat{T}$ the length of the period of time before the investor receives a preference shock. By definition, $\hat{T}$ is exponentially distributed with mean $1/\delta$. The value of an investor can then be written recursively as follows,

$$\tilde{V}_i(a)=\mathbb{E}\left[\int_0^{\tilde{T}\wedge\hat{T}} e^{-rs}u_i(a)ds+\mathbb{I}_{\left\{\hat{T}<\tilde{T}\right\}}e^{-r\hat{T}}\tilde{V}_{k(\hat{T})}(a)\right], \tag{34}$$

where $k(\hat{T})$ indicates the new realization of the preference shock at time $\hat{T}$. Using the fact that $\tilde{T}$ and $\hat{T}$ are independent random variables, one can rewrite the first term on the right-hand side of (34) as

$$\begin{aligned}\mathbb{E}\left[\int_0^{\tilde{T}\wedge\hat{T}} e^{-rs}u_i(a)ds\right] &= \mathbb{E}\left[\int_0^{\infty}\mathbb{I}_{\{s\leq\tilde{T}\wedge\hat{T}\}}e^{-rs}u_i(a)\,ds\right]=u_i(a)\int_0^{\infty}\mathbb{E}\left[\mathbb{I}_{\{s\leq\tilde{T}\wedge\hat{T}\}}\right]e^{-rs}\,ds\\ &= u_i(a)\int_0^{\infty}e^{-(r+\kappa+\delta)s}\,ds=\frac{u_i(a)}{r+\kappa+\delta}.\end{aligned} \tag{35}$$

The second equality follows because $u_i(a)$ is constant over the interval of integration, and by interchanging the integral and expectation sign. The third equality follows because $\tilde{T}$ and $\hat{T}$ are independent exponential random variables with respective parameter κ and δ: thus $\tilde{T}\wedge\hat{T}$ is exponential as well with parameter $\kappa+\delta$.

Turning to the second term in (34), we first note that the realizations of the preference shocks are independent and identically distributed according to π_i. Thus, the distribution of $k(\hat{T})$ is given by $\{\pi_i\}_{i=1}^I$. Therefore,

$$\mathbb{E}\left[\mathbb{I}_{\left\{\hat{T}<\tilde{T}\right\}}e^{-r\hat{T}}\tilde{V}_{k(\hat{T})}(a)\right] = \mathbb{E}\left[\mathbb{I}_{\left\{\hat{T}<\tilde{T}\right\}}e^{-r\hat{T}}\right]\sum_{k=1}^{I}\pi_k\tilde{V}_k(a)=\frac{\delta}{\delta+r+\kappa}\sum_{k=1}^{I}\pi_k\tilde{V}_k(a). \tag{36}$$

Adding (35)and (36), one finds

$$\tilde{V}_i(a)=\frac{u_i(a)}{r+\kappa+\delta}+\frac{\delta}{r+\kappa+\delta}\sum_{k=1}^{I}\pi_k\tilde{V}_k(a), \tag{37}$$

for all $i\in\{1,\ldots,I\}$. One then easily verifies that this system of equation is solved by

$$\tilde{V}_i(a)=\frac{U_i\left(a\right)}{r+\kappa}, \tag{38}$$

where $U_i\left(a\right)$ is as in (8).

To derive expression (9), just note that the expected discounted price at the time the investor regains direct access to the asset market is:

$$\mathbb{E}[e^{-r\tilde{T}}p(t+\tilde{T})]=\kappa\int_0^{\infty}e^{-(r+\kappa)s}p(t+s)ds. \tag{39}$$

A.2.4 The expression for $U_i(a)$ and $\xi(t)$ during the crisis

We let

$$
\begin{aligned}
\tilde{V}_i^C(a) &= \mathbb{E}\left[\int_0^{\tilde{T}} e^{-rs}\left(\theta + (1-\theta)\mathbb{I}_{\{s\geq T_\rho\}}\right) u_i(a)\, ds\right] \\
&= \mathbb{E}\left[\int_0^{\tilde{T}\wedge\hat{T}\wedge T_\rho} e^{-rs}\theta u_i(a)\, ds\right] + \mathbb{E}\left[\mathbb{I}_{\{\hat{T}<\tilde{T}\wedge T_\rho\}} e^{-r\hat{T}}\tilde{V}^C_{k(\hat{T})}(a)\right] + \mathbb{E}\left[\mathbb{I}_{\{T_\rho<\tilde{T}\wedge\hat{T}\}} e^{-rT_\rho}\tilde{V}_i(a)\right] \\
&= \frac{\theta u_i(a)}{r+\kappa+\delta+\rho} + \frac{\delta}{r+\kappa+\delta+\rho}\sum_{j=1}^{I}\tilde{V}_j^C(a) + \frac{\rho}{r+\kappa+\delta+\rho}\tilde{V}_i(a) \\
&= \frac{\theta u_i(a) + \rho\tilde{V}_i(a)}{r+\kappa+\delta+\rho} + \frac{\delta}{r+\kappa+\delta+\rho}\sum_{j=1}^{I}\tilde{V}_j^C(a).
\end{aligned}
$$

where the second last equality follows from the exact same calculation as for $\tilde{V}_i(a)$ in the previous paragraph. One sees that this is exactly the same equation as (37), except that $u_i(a)$ is replaced by $\theta u_i(a) + \rho\tilde{V}_i(a)$ and κ is replaced by $\kappa + \rho$. Thus the result of the last section applies and we have that:

$$
(r+\kappa+\rho)\tilde{V}_i^C(a) = \frac{(r+\kappa+\rho)\left(\theta u_i(a) + \rho\tilde{V}_i(a)\right) + \delta\sum_{j=1}^{I}\left(\theta u_j(a) + \rho\tilde{V}_j(a)\right)}{r+\kappa+\rho+\delta}
$$

Letting $U_i^C(a) = (r+\kappa)\tilde{V}_i^C(a)$, we have

$$
\begin{aligned}
U_i^C(a) &= \frac{r+\kappa}{r+\kappa+\rho}(r+\kappa+\rho)\tilde{V}_i^C(a) \\
&= \frac{r+\kappa}{r+\kappa+\rho}\,\frac{(r+\kappa+\rho)\left(\theta u_i(a) + \rho\tilde{V}_i(a)\right) + \delta\sum_{j=1}^{I}\left(\theta u_j(a) + \rho\tilde{V}_j(a)\right)}{r+\kappa+\rho+\delta} \\
&= \frac{(r+\kappa+\rho)\left[\frac{r+\kappa}{r+\kappa+\rho}\theta u_i(a) + \frac{\rho}{r+\kappa+\rho}U_i(a)\right] + \delta\sum_{j=1}^{I}\pi_j\left[\frac{r+\kappa}{r+\kappa+\rho}\theta u_j(a) + \frac{\rho}{r+\kappa+\rho}U_j(a)\right]}{r+\kappa+\rho+\delta},
\end{aligned}
$$

keeping in mind that $U_i^C(a) = (r+\kappa)\tilde{V}_i(a)$. This is the formula stated in the Lemma.

To derive the expected value of the re-sale price, we use the fact that $\tilde{T} - t$ and $T_\rho - t$ are two independent exponentially distributed random variables:

$$
\begin{aligned}
&\mathbb{E}\left[e^{-r(\tilde{T}-t)}\left[\mathbb{I}_{\{\tilde{T}<T_\rho\}}p^C(\tilde{T}) + \mathbb{I}_{(\tilde{T}\geq T_\rho)}p(\tilde{T}\,|\,T_\rho)\right]\right] = \\
&\int_t^\infty\int_t^\infty e^{-r(\tau_\kappa-t)}\left[\mathbb{I}_{\{\tau_\kappa<\tau_\rho\}}p^C(\tau_\kappa) + \mathbb{I}_{\{\tau_\kappa\geq\tau_\rho\}}p(\tau_\kappa\,|\,\tau_\rho)\right]\kappa e^{-\kappa(\tau_\kappa-t)}\rho e^{-\rho(\tau_\rho-t)}\,d\tau_\rho\,d\tau_\kappa \\
&= \int_t^\infty e^{-r(\tau_\kappa-t)}\left[e^{-\rho(\tau_\kappa-t)}p^C(\tau_\kappa) + \int_t^{\tau_\kappa}\rho e^{-\rho(\tau_\rho-t)}p(\tau_\kappa\,|\,\tau_\rho)\right]\kappa e^{-\kappa(\tau_\kappa-t)}\,d\tau_\rho\,d\tau_\kappa.
\end{aligned}
$$

A.3 Proof of Proposition 1

Suppose $A_d(t) > 0$ for some $t \geq T_\rho$. Let $T = \inf\{s \geq t : A_d(s) = 0\}$. Since $A_d(s)$ is continuous, we have $A_d(T) = 0$ and so $T > t$. Now for $s \in [t, T)$, $A_d(s) > 0$ so $A_d(s)$ and $\xi(s)$ solve the system of ODEs given by (13) and

$$
\dot{\xi}(s) = (r+\kappa)\xi(s).
$$

Integrating the second ODE gives $\xi(s) = \xi(t)e^{(r+\kappa)(s-t)}$. Plugging this back into the first ODE, (13), gives:

$$
A_d(s) = e^{-\alpha(s-t)}A_d(t) + \alpha\int_t^s e^{-\alpha(s-u)}\left[A - \sum_{i=1}^{I}\pi_i U_i'^{-1}\left[\xi(u)\right]\right]du. \tag{40}
$$

Equipped with this equation, we first prove:

Lemma 7. *If, for some $t \geq T_\rho$, $A_d(t) > 0$, then $\xi(t) < \bar{\xi}$ and $\dot{A}_d(t) < 0$.*

Suppose to the contrary that there were some $t \geq T_\rho$ such that $A_d(t) > 0$ and $\xi(t) \geq \bar{\xi}$. Then, the above calculation show that $\xi(u) \geq \bar{\xi}$ for all $u \in [t, T)$, and therefore:

$$A - \sum_{i=1}^{I} \pi_i U_i'^{-1}\left[\xi(u)\right] \geq A - \sum_{i=1}^{I} \pi_i U_i'^{-1}\left[\bar{\xi}\right] = 0.$$

It thus follow that $A_d(s) > e^{-\alpha(s-t)} A_d(t)$. Since $A_d(T) = 0$ we must have that $T = \infty$. But this means that $\dot{\xi}(s) = (r+\kappa)\xi(s)$ for all $s \geq t$ and, because of equation (11), that $rp(s) = \dot{p}(s)$ for all $s \geq t$. But then the only way the no-bubble condition holds is if $p(t) = \xi(t) = 0$, which is impossible given that $\xi(t) \geq \bar{\xi}$. That $\dot{A}_d(t) < 0$ follows from substituting $\xi(t) < \bar{\xi}$ in ODE (13).

We are now ready to solve for an equilibrium path. We start at T_ρ with some positive inventory $A_d(T_\rho) > 0$. Let T be the first time greater than T_ρ such that $A_d(T) = 0$. If $T = \infty$, then as before the no-bubble condition would be violated. So $T < \infty$. Since $A_d(t) > 0$ for all $t < T$, it follows by Lemma 7 and the continuity of $\xi(t)$ that $\xi(T) \leq \bar{\xi}$. But if $\xi(T) < \bar{\xi}$ then ODE (13) implies that $\dot{A}_d(T) < 0$. Moreover, since $\xi(s)$ is continuous, it follows from ODE (13) that $A_d(t)$ is continuously differentiable. Thus, we must have that $\dot{A}_d(s) < 0$ for some $s > T$, which would violate the short-selling constraint. Therefore, $\xi(T) = \bar{\xi}$. Next, we show that $A_d(s) = 0$ for all $s \geq T$. Suppose to the contrary that there is some $s > T$ such that $A_d(s) > 0$. Since $A_d(t)$ is continuously differentiable, we can apply Taylor Theorem and find some $s' \in [T, s]$ such that

$$\dot{A}_d(s') = \frac{A_d(s) - A_d(T)}{s - T} > 0.$$

The contrapositive of Lemma 7 then implies that $A_d(s') = 0$. Now, since $A_d(s')$ is continuously differentiable, there must be some $u > s'$ such that $A_d(u) > 0$ and $\dot{A}_d(u) > 0$, which contradicts Lemma 7. Thus $A_d(s) = 0$ for all $s \geq T$. Plugging this back into equation (13), it follows that $\xi(s) = \bar{\xi}$ for all $s \geq T$.

To solve for T, we plug $\xi(t) = \bar{\xi} e^{-(r+\kappa)(T-t)}$ back into equation (40) and solve for the unique solution of $A_d(T) = 0$, given the initial condition $A_d(T_\rho)$. That is, one has to solve the equation:

$$\begin{aligned}
& 0 = e^{-\alpha(T-T_\rho)} A_d(T_\rho) + \alpha \int_{T_\rho}^{T} e^{-\alpha(T-u)} \left[A - D(\bar{\xi} e^{-(r+\kappa)(T-u)})\right] \\
\Leftrightarrow \quad & 0 = A_d(T_\rho) + \alpha \int_{T_\rho}^{T} e^{-\alpha(T_\rho-u)} \left[A - D(\bar{\xi} e^{-(r+\kappa)(T-u)})\right] \\
\Leftrightarrow \quad & 0 = A_d(T_\rho) + \alpha \int_{0}^{\Delta T} e^{\alpha s} \left[A - D(\bar{\xi} e^{-(r+\kappa)(\Delta T-s)})\right], \qquad (41)
\end{aligned}$$

where

$$D(\xi) \equiv \sum_{i=1}^{I} \pi_i U_i'^{-1}\left[\xi(u)\right], \qquad (42)$$

where the first equivalence follows from multiplying through by $e^{\alpha(T-T_\rho)}$, and the second one from the change of variable $\Delta T \equiv T - T_\rho$ and $s = u - T_\rho$. Since the function $D(\xi)$ is decreasing and since $D(\bar{\xi} e^{-(r+\kappa)(\Delta T-s)}) > D(\bar{\xi}) = A$ for all $s < \Delta T$, it follows that the right-hand side of (41) is a strictly increasing function of $A_d(T_\rho)$ and a strictly decreasing function of ΔT. Since $A_d(T_\rho) > 0$, it is clearly strictly positive at $\Delta T = 0$. Moreover, since $D(\xi)$ is strictly decreasing and since $A - D(\bar{\xi} e^{-(r+\kappa)(\Delta T-s)}$ is negative, we have

$$\begin{aligned}
& \alpha \int_{0}^{\Delta T} e^{\alpha s} \left[A - D(\bar{\xi} e^{-(r+\kappa)(\Delta T-s)})\right] \\
\leq \quad & \alpha \int_{0}^{\Delta T > \varepsilon} e^{\alpha s} \left[A - D(\bar{\xi} e^{-(r+\kappa)\varepsilon})\right] \\
= \quad & e^{\alpha(\Delta T-\varepsilon)} \left[A - D(\bar{\xi} e^{-(r+\kappa)\varepsilon})\right] \to -\infty,
\end{aligned}$$

as ΔT goes to infinity. Thus, equation (41) has a unique solution $\Delta T > 0$ and an application of the implicit function theorem shows that it strictly increasing in $A_d(T_\rho)$ and twice continuously differentiable. Moreover, it goes to infinity as $A_d(T_\rho)$ goes to infinity. Indeed since it is a monotonic function, it must have a limit. This limit can't be finite: otherwise, the second term on the right-hand-side of (41) would go to some finite limit, which is impossible since the first term goes to infinity and the two terms must sum to zero. From ΔT we obtain the function $\psi(A_d) = \bar{\xi} e^{-(r+\kappa)\Delta T}$. Therefore:

Lemma 8. *The function $\psi(A_d)$ is strictly decreasing, twice continuously differentiable, and goes to zero as A_d goes to infinity.*

A.4 Proof of Lemma 3

We proceed as in the Proof of Lemma 1, but for the integration by part we break the interval of integration in two: $[0, T_\rho \wedge T]$ and $[T_\rho \wedge T, T]$. After using the no-bubble condition, we find that the value of the inventory path is:

$$\begin{aligned}
& a_d(t)p^C(t) + \int_t^{T_\rho} e^{-r(s-t)} \left(\dot{p}^C(s) - rp^C(s)\right) a_d^C(s)\, ds \\
& + e^{-r(T_\rho - t)} \left(p(T_\rho \,|\, T_\rho) - p^C(t)\right) a_d^C(T_\rho) \\
& + \int_{T_\rho}^{\infty} e^{-r(s-t)} \left(\frac{\partial p}{\partial t}(t \,|\, T_\rho) - rp(t \,|\, T_\rho)\right) a_d(t \,|, T_\rho)\, ds.
\end{aligned}$$

Taking expectations, ignoring the initial condition $a_d(t)p^C(t)$ and the last term that only depends on the inventory plan $a_d(T_\rho, t)$ along the recovery path, we find that before T_ρ the dealer chooses $a_d^C(s)$ in order to maximize:

$$\begin{aligned}
& \mathbb{E}_t \left[\int_t^{\infty} \mathbb{I}_{\{s \leq T_\rho\}} e^{-r(s-t)} \left(\dot{p}^C(s) - rp^C(s)\right) a_d^C(s)\, ds\right] \\
& + \mathbb{E}_t \left[e^{-r(T_\rho - t)} \left(p(T_\rho \,|\, T_\rho) - p^C(t)\right) a_d^C(T_\rho)\right].
\end{aligned}$$

Note that, in the first expectation, the only random variable is $\mathbb{I}_{\{t \leq T_\rho\}}$ and its expectation is $e^{\rho(s-t)}$ for each s. Next, write the second expectation as an integral against the exponential density $\rho e^{\rho(s-t)}$. After collecting terms, we find that the dealer's objective is:

$$\int_t^{\infty} e^{-(r+\rho)(s-t)} \left[\dot{p}^C(s) - rp^C(s) + \rho\left(p(T_\rho, T_\rho) - p^C(t)\right)\right] a_d^C(s)\, ds,$$

and we can apply the same argument as in Lemma 1.

A.5 Proof of Proposition 2

The system of ODE we seek to solve is:

$$\dot{\xi}^C(t) = (r + \rho + \kappa)\xi^C(t) - \rho\psi(A_d^C(t)) \tag{43}$$

$$\dot{A}_d^C(t) = \alpha\left[A - A_d^C(t) - D^C(\xi^C(t))\right] \tag{44}$$

where $D^C(\xi)$ is defined as in equation (42) but based on $U_i^C(a)$. Given Lemma 8 and under our maintained regularity assumptions on the utility functions, we can apply standard existence and uniqueness Theorems for ODEs (see, for example, Theorem 6.2.3 in Hubbard and West, 1995) given the initial condition $A_d^C(0) = 0$ and $\xi^C(0) > 0$. As it is standard with forward-looking rational expectations dynamics, the initial condition $\xi^C(0)$ is found by arguing that the economy has to evolve along a saddle path of the dynamic system (43)-(44). Precisely, we establish two results: in Section A.5.1, we show that there exists a unique saddle path extending from the steady state $(\bar{\xi}^C, \bar{A}_d^C)$ to some initial condition $A_d^C(0) = 0$ and $\bar{\xi}^C(0) > 0$. Second, in Section A.5.2, we argue that other paths can't be the basis of an equilibrium.

A.5.1 The unique saddle path

We already established in the text that there is a unique steady state. Next, we verify that it has the local saddle-point property: the Jacobian of the system of differential equation at $(\bar{A}_d^C, \bar{\xi}^C)$ has two real eigenvalues which have opposite sign. The Jacobian is

$$\begin{pmatrix} (r + \rho + \kappa) & -\rho\psi'(\bar{A}_d^C) \\ -\alpha D^{C\prime}(\bar{\xi}^C) & -\alpha \end{pmatrix}.$$

Clearly, the determinant of the Jacobian is strictly negative which for a 2-by-2 matrix means that the matrix has two real eigenvalues with opposite signs. We can then apply Theorem 8.3.2 in Hubbard and West (1995) to assert that there is a unique trajectory that tends to $(\bar{A}_d^C, \bar{\xi}^C)$ from the left. This saddle path is indicated by the plain curve with double arrow in Figure 8.

Next, we need to show that this saddle path can be extended back to the y-axis, delivering the initial condition $\xi^C(0)$. We proceed in two steps. First we argue that, as long as $A_d^C(t) \geq 0$, the saddle path has to remain trapped into the area denoted by K and shaded in the figure, i.e. the area delimited by the y-axis to the west, the isocline $\dot{\xi}^C(t) = 0$ to the north, and the isocline $\dot{A}_d^C(t) = 0$ to the south. We know that the saddle path must eventually lie in K. Let t_1 be the last time when the saddle path enters K from outside. After t_1, the saddle path stays in K and converges to the steady state $(\bar{A}_d^C, \bar{\xi}^C)$. When the saddle path is in K, $A_d^C(t)$ increases and $\xi^C(t)$ decreases. Therefore, we have $A_d^C(t_1) < \bar{A}_d^C$ and $\xi^C(t_1) > \bar{\xi}^C$. Suppose that, at t_1, the saddle path enters K from the north, crossing the isocline $\dot{\xi}^C(t_1) = 0$ from above. Differentiating ODE (43) yields:

$$\ddot{\xi}^C(t) = (r + \rho + \kappa)\dot{\xi}^C(t_1) - \rho\psi'(A_d^C(t_1))\dot{A}_d^C(t_1) = -\rho\psi'(A_d^C(t_1))\dot{A}_d^C(t_1) > 0.$$

since $\psi'(A) < 0$ and $\dot{A}_d^C(t_1) > 0$ because $A_d^C(t_1) < \bar{A}_d^C$ and lies above the isocline $\dot{A}_d^C(t) = 0$. Thus, just after t_1, $\dot{\xi}^C(t)$ is strictly positive. But this is a contradiction: since the saddle path enters K from the north, at time t_1 $\dot{\xi}^C(t)$ must move from being zero to being strictly negative. Alternatively the saddle path cannot enter K from the south, because i) at that time $\xi(t)$ would have a value less than the steady state and ii) once the saddle path enters K for the last time, $\xi(t)$ is decreasing.

Now let us start the system on the saddle path with an initial condition to the left of the steady state, say $\tilde{A}_d^C(t_0)$ and $\tilde{\xi}^C(t_0)$, and let us run the system backward in time, for $t_0 - s \leq t_0$ (formally, this means making the change of variable $u = t_0 - s$ in the system of ODEs (43) and (44)). Graphically, think of moving along the saddle path towards the northwest of Figure 8. Since the saddle path stays in K, we know that $\tilde{A}_d^C(t_0 - s)$ is decreasing in s. Moreover, note that $\tilde{\xi}^C(t_0 - s) > \tilde{\xi}^C(t_0)$ and that $\tilde{A}_d^C(t_0 - s) < \tilde{A}_d^C(t_0)$. Plugging this back into ODE (44), we find that

$$\begin{aligned}
\frac{d\tilde{A}_d^C(t_0 - s)}{ds} &= -\dot{\tilde{A}}_d^C(t_0 - s) = -\alpha\left(A - \tilde{A}_d^C(t_0 - s) - D^C(\tilde{\xi}^C(t_0 - s))\right) \\
&\leq -\alpha\left(A - \tilde{A}_d^C(t_0) - D(\tilde{\xi}^C(t_0))\right) = -\alpha\dot{\tilde{A}}_d^C(t_0) < 0.
\end{aligned}$$

So the derivative of $\tilde{A}_d^C(t_0 - s)$ is negative and bounded away from zero, implying that $\tilde{A}_d^C(t_0 - s)$ reaches zero in finite time, say at s_0. This proves that the saddle path extends to the y-axis, and delivers the initial condition $\xi^C(0) = \tilde{\xi}^C(t_0 - s_0)$.

A.5.2 Ruling out other solutions

Next, we need to show that other solutions of the system (43)-(44) can't be the basis of an equilibrium.

Preliminary remarks. Let J be the region of the positive quadrant below both isoclines and such that $A_d^C < \bar{A}_d^C$. Similarly, let L be the region of the positive quadrant above both isocline. The argument that allowed us to conclude that the saddle path stays trapped in region K also shows that a solution of the system can only move from region K to region L, and not vice versa. Thus, once a solution leaves K to L, it never comes back to K. One easily shows (using the same argument) that a solution can never leave region L. Similarly, one can show that a solution can only move from region K to region J, and not vice versa.

Now consider alternative initial conditions for $\xi^C(0)$. We let ξ_1 (ξ_2) denote the intersection of the $\dot{A}_d^C(t) = 0$ ($\dot{\xi}^C(t) = 0$) isocline with the y-axis. The condition for the existence of a steady state with $\bar{A}_d^C > 0$ implies that $\xi_1 \leq \xi_2$.

An initial condition $\xi^C(0) < \xi^1$. This can't be the basis of an equilibrium because $\dot{A}_d^C(0) < 0$: given that $A_d^C(0) = 0$, this would violate the dealers' short-selling constraint.

An initial condition $\xi^C(0) \in [\xi_1, \xi_2]$. Suppose there is a candidate equilibrium path with an initial condition in $[\xi_1, \xi_2]$ that is different from that of the saddle path. Because solutions of ODEs never cross, this candidate equilibrium path remains different from the saddle path at all subsequent times. Then we claim that the equilibrium path would eventually leave region K. Suppose that it stayed in K: then $A_d^C(t)$ would be increasing and bounded above and $\xi^C(t)$ would be is decreasing, so this candidate equilibrium path would have a limit in K as $t \to \infty$. But the limit must be equal to the unique steady state of the model which is impossible because this candidate equilibrium path is different from the saddle path. Thus, this we have two possibilities.

If the equilibrium leaves region K for region J at some time t, then it is clear from the figure that $A_d^C(t) < \bar{A}_d$. We also know from the previous paragraph that it never re-enters region K. Given that $\dot{A}_d^C(t) < 0$ in J, and given the equilibrium restrictions that $A_d^C(t) \geq 0$ and $\xi^C(t) \geq 0$, we obtain that the equilibrium path must stay trapped in region J forever, with $A_d^C(s) \leq A_d^C(t) < \bar{A}_d^C$ for all $s \geq t$. Using the fact that $A_d^C(t)$ and $\xi^C(t)$ are decreasing in J and bounded below by zero, we obtain that the candidate equilibrium path has a finite limit as $t \to \infty$, with $A_d^C(\infty) < \bar{A}_d^C$. But this is impossible given that $(\bar{A}_d^C, \bar{\xi}^C)$ is the unique steady state of the system.

If the equilibrium leaves region K for region L then we know it remains in L forever after. Starting from some time t when the candidate equilibrium is away from the boundary of L, we can solve ODE (43) given the path for $A_d^C(t)$:

$$\begin{aligned}
\xi^C(s) &= \xi^C(t)e^{(r+\rho+\kappa)(s-t)} - \int_t^s \rho\psi(A_d^C(u))e^{-(r+\rho+\kappa)(u-s)}\,du \\
&\geq \xi^C(t)e^{(r+\rho+\kappa)(s-t)} - \rho\psi(A_d^C(t))\int_t^s e^{-(r+\rho+\kappa)(u-s)}\,du \\
&= \left(\xi^C(t) - \frac{\rho\psi(A_d^C(t))}{r+\rho+\kappa}\right)e^{(r+\rho+\kappa)(s-t)} + \frac{\rho\psi(A_d^C(t))}{r+\rho+\kappa}.
\end{aligned}$$

where the second inequality follows because $A_d^C(t)$ is increasing since the equilibrium stays in L, and the third equality from integrating. Because we start away from the boundary we have that $\dot{\xi}^C(t) = (r+\rho+\kappa)\xi^C(t) - \rho\psi(A_d^C(t)) > 0$, so it follows from the above formula that $\xi^C(s)$ grows towards infinity at rate $r+\rho+\kappa$. But

$$p^C(s)e^{-rs} \quad \geq \quad p^C(s)e^{-rs} - \mathbb{E}_s\left[p(\tilde{T})e^{-r\tilde{T}}\right] = \xi^C(s)e^{-rs}.$$

So $p^C(s)e^{-rs}$ goes to infinity given that $\xi^C(s)$ grows at a rate $r+\rho+\kappa > r$. Thus the no-bubble condition is violated, and this rule out this candidate equilibrium path.

An initial condition $\xi^C(0) > \xi_2$. Then the equilibrium path starts in region L so the reasoning of the previous paragraph implies that the no-bubble condition is violated.

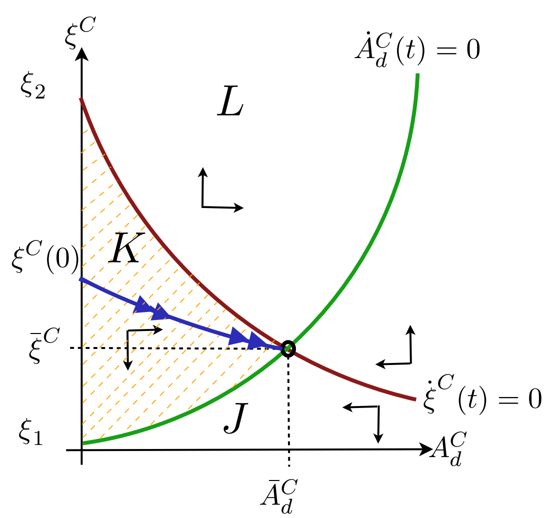

Figure 8: Phase diagram

A.6 Proof of Proposition ??

The price goes up during the recovery because it solves the ODE $\dot{p}(t) = rp(t)$ so it is equal to $p(t) = p(T_\rho)e^{r(t-T_\rho)}$, which is an increasing function of time. Before the recovery, the price solves the ODE:

$$\dot{p}^C(t) = (r+\rho)p^C(t) - \rho p(t \,|\, t). \tag{45}$$

Note that $p(t \,|\, t) = e^{-r(T-t)}\bar{\xi}/r$, where T denotes the time at which dealers have unwound their inventories and the price has reached its steady-state value. By definition of T and $\psi(A_d(t)^C)$, we have

$$\psi(A_d(t)^C) = e^{-(r+\kappa)(T-t)}\bar{\xi},$$

implying that:

$$p(t \,|\, t) = \left[\psi(A_d(t)^C)\right]^{\frac{r}{r+\kappa}} \bar{\xi}^{\frac{\kappa}{r+\kappa}}.$$

Since $A_d^C(t)$ is increasing and $\psi(A)$ is decreasing, it follows that $p(t,t)$ is decreasing. Now integrating (45) and using the no-bubble condition, it follows that:

$$p^C(t) = \int_t^\infty e^{-(r+\rho)(s-t)} \rho p(s \,|\, s)\, ds < \frac{\rho p(t \,|\, t)}{r+\rho},$$

because $p(s \,|\, s) < p(t \,|\, t)$. Note that this implies in particular that $p^C(t) < p(t \,|\, t)$: at the recovery time, the price jumps up. Rearranging this inequality gives $(r+\rho)p^C(t) - p(t \,|\, t) < 0$ and comparing with (45) yields $\dot{p}^C(t) < 0$.

A.7 Proof of Proposition 3

Consider first the case when $\kappa \to \infty$. Then, one sees that $\bar{\varepsilon}_i \to \varepsilon_i$ while $\bar{\varepsilon}_i^C \to \theta\varepsilon_i$. Thus, the left-hand side of condition (30) converges to $\theta^{1/\sigma}$. The right-hand side, on the other hand, converges to zero. Therefore, the condition is satisfied and dealers accumulate no inventories.

Let us turn to the case $r+\kappa \to 0$. Then, one sees easily that both sides of (30) go to 1. Therefore, in order to figure out the direction of the inequality, we need a first-order Taylor expansion of both sides as $r+\kappa \to 0$. To simplify the algebra, let us define $\gamma \equiv r+\kappa$ and let us normalize $\sum_{j=1}^I \pi_j\varepsilon_j = 1$. Then, we have:

$$\bar{\varepsilon}_i = \frac{\gamma\varepsilon_i + \delta}{\delta+\gamma} = 1 + \frac{\gamma}{\delta+\gamma}(\varepsilon_i - 1) = 1 + \frac{\gamma}{\delta}(\varepsilon_i - 1) + o(\gamma),$$

where $o(\gamma)$ is a function such that $o(\gamma)/\gamma \to 0$ as $\gamma \to 0$. It follows that:

$$\bar{\varepsilon}_i^{\frac{1}{\sigma}} = 1 + \frac{\gamma}{\delta\sigma}(\varepsilon_i - 1) + o(\gamma)$$

Keeping in mind that $\sum_{j=1}^I \pi_j\varepsilon_j = 1$, we obtain:

$$\sum_{j=1}^I \pi_j \left(\bar{\varepsilon}_j\right)^{\frac{1}{\sigma}} = 1 + o(\gamma). \tag{46}$$

Next, we have:

$$\begin{aligned}
\varepsilon_i^C &= \frac{\gamma\theta\varepsilon_i + \rho\bar{\varepsilon}_i}{\gamma+\rho} = (\gamma+\rho)^{-1}\left[\gamma\theta(\varepsilon_i - 1) + \gamma(\theta-1) + \gamma + \rho + \frac{\rho\gamma}{\delta}(\varepsilon_i - 1) + o(\gamma)\right] \\
&= 1 + \frac{\gamma}{\gamma+\rho}\left[\left(\theta + \frac{\rho}{\delta}\right)(\varepsilon_i - 1) + (\theta - 1) + o(1)\right] = 1 + \gamma\left(\frac{\theta}{\rho} + \frac{1}{\delta}\right)(\varepsilon_i - 1) + \frac{\gamma}{\rho}(\theta - 1) + o(\gamma)
\end{aligned}$$

Therefore:

$$\begin{aligned}
\bar{\varepsilon}_i^C &= \frac{(\gamma+\rho)\varepsilon_i^C + \delta\sum_{j=1}^I \pi_j\varepsilon_j}{\gamma+\rho+\delta} = (\gamma+\rho+\delta)^{-1}\left[(\gamma+\rho+\delta)\left(1 + \frac{\gamma}{\rho}(\theta-1)\right) + (\gamma+\rho)\gamma\left(\frac{\theta}{\rho} + \frac{1}{\delta}\right)(\varepsilon_i - 1) + o(\gamma)\right] \\
&= 1 + \frac{\gamma}{\rho}(\theta - 1) + \frac{\gamma}{\delta}\frac{\theta\delta + \rho}{\delta+\rho}(\varepsilon_i - 1) + o(\gamma).
\end{aligned}$$

Now this implies that:

$$\left(\bar{\varepsilon}_i^C\right)^{\frac{1}{\sigma}} = 1 + \frac{\gamma}{\rho\sigma}(\theta - 1) + \frac{\gamma}{\delta\sigma}\frac{\theta\delta + \rho}{\delta + \rho} + o(\gamma).$$

As before, keeping in mind that $\sum_{j=1}^{I} \pi_j \varepsilon_j = 1$, this gives:

$$\sum_{j=1}^{I} \pi_j \left(\bar{\varepsilon}_j^C\right)^{\frac{1}{\sigma}} = 1 + \frac{\gamma}{\rho\sigma}(\theta - 1) + o(\gamma) \tag{47}$$

Taken together, equations (46) and (47) show that the left-hand side of (30) is equal to:

$$1 + \frac{\gamma}{\rho\sigma}(\theta - 1) + o(\gamma)$$

The right-hand side of (30) is, on the other hand:

$$\left(\frac{\rho}{\rho + \gamma}\right)^{\frac{1}{\sigma}} = 1 - \frac{\gamma}{\rho\sigma} + o(\gamma).$$

Comparing the left-hand side with the right-hand side, it is clear, then, that condition (30) is satisfied for γ close enough to zero.

A.8 Proof of Proposition 4

The planner's problem can be described recursively as follows. Following the recovery, the maximum attainable welfare for society is

$$\begin{aligned}
\mathcal{W}(A_d) &= \max_{\{a_i(t)\}_{i=1}^{I}, A_d(t)} \alpha \int_0^\infty \frac{\sum_{i=1}^{I} \pi_i \hat{\bar{\varepsilon}}_i u(a_i(t))}{r + \alpha} e^{-rt}\, dt \\
\text{s.t.} \quad \dot{A}_d(t) &= \alpha \left\{ A - A_d(t) - \sum_{i=1}^{I} \pi_i a_i(t) \right\} \\
A_d(0) &= A_d.
\end{aligned}$$

Let $\lambda(t)$ be the current-valued costate variable associated with $A_d(t)$. From the Maximum Principle, the necessary conditions for an optimum are

$$\begin{aligned}
\frac{\hat{\bar{\varepsilon}}_i u'(a_i(t))}{r + \alpha} - \lambda(t) &\leq 0, \quad \text{“ = ” if } a_i(t) > 0, \\
\dot{\lambda}(t) - (r + \alpha)\lambda(t) &\leq 0, \quad \text{“ = ” if } A_d(t) > 0.
\end{aligned}$$

The Mangasarian sufficient condition is $\lim_{t\to\infty} e^{-rt}\lambda(t)A_d(t) = 0$. These conditions coincide with the equilibrium conditions if and only if $\eta = 0$.

Before the recovery, and using the fact that T_ρ is exponentially distributed with parameter ρ, the planner's problem is

$$\begin{aligned}
\mathcal{W}^C(A_d^C) &= \int_0^\infty e^{-(r+\rho)t} \left(\frac{\alpha \sum_{i=1}^{I} \pi_i \hat{\bar{\varepsilon}}_i^C u(a_i^C(t))}{r + \alpha} + \rho \mathcal{W}(A_d^C(t)) \right) dt. \\
\text{s.t.} \quad \dot{A}_d^C(t) &= \alpha \left\{ A - A_d^C(t) - \sum_{i=1}^{I} \pi_i a_i^C(t) \right\} \\
A_d^C(0) &= A_d^C.
\end{aligned}$$

Let $\lambda^C(t)$ be the current-valued costate variable associated with $A_d^C(t)$. From the Maximum Principle, and using that $\mathcal{W}'(A_d^C(t)) = \lambda(0; A_d^C(t))$, the necessary conditions for an optimum are

$$\begin{aligned}
\frac{\hat{\bar{\varepsilon}}_i^C u'(a_i(t))}{r+\alpha} - \lambda^C(t) &\leq 0, \quad \text{“}=\text{”} \text{ if } a_i^C(t) > 0, \\
\dot{\lambda}^C(t) + \rho\left[\lambda(0; A_d^C(t)) - \lambda^C(t)\right] - (r+\alpha)\lambda^C(t) &\leq 0, \quad \text{“}=\text{”} \text{ if } A_d^C(t) > 0.
\end{aligned}$$

The Mangasarian sufficient condition is $\lim_{t\to\infty} e^{-(r+\rho)t}\lambda^C(t)A_d^C(t) = 0$. These conditions coincide with the equilibrium conditions if and only if $\eta = 0$.

The second part of the Proposition is a consequence of Proposition 3.

A.9 Proof of Proposition 6

The welfare criterion is that of equation (4.2).

Welfare during the crisis. We first evaluate the first integral in the expectation. First, recall that during the crisis the government intervention amounts to scale down the available supply in the market by a factor $1-\omega$. Because of iso-elastic utilities, all investors' holdings are scaled down by that same factor. Thus, if we let a_i^C be an investor's holding during the crisis in the absence of government intervention, we find that welfare during the crisis with government intervention is equal to:

$$\frac{\alpha}{r+\alpha}\int_0^{T_\rho}\sum_i \pi_i \hat{\bar{\varepsilon}}_i^C \frac{{a_i^C}^{1-\sigma}}{1-\sigma}(1-\omega)^{1-\sigma}e^{-rt}\,dt = \frac{\alpha}{r+\alpha}\frac{1-e^{-rT_\rho}}{r}\sum_i \pi \hat{\bar{\varepsilon}}_i^C \frac{{a_i^C}^{1-\sigma}}{1-\sigma}(1-\omega)^{1-\sigma}.$$

Now recall that, when ω is close to zero, we have that $(1-\omega)^{1-\sigma} = 1-(1-\sigma)\omega + o(\omega)$. Plugging this back in the expression above, we find that the *change* in welfare during the crisis is equal to:

$$-\omega\frac{\alpha}{r+\alpha}\frac{1-e^{-rT_\rho}}{r}\sum_i \pi\hat{\bar{\varepsilon}}_i^C {a_i^C}^{1-\sigma} + o(\omega). \tag{48}$$

Welfare during the recovery. Suppose the recovery starts are time T_ρ, with government holdings equal to $A_g(T_\rho) = \omega(1-e^{-\alpha T_\rho})$. Then, from time T_ρ to some time T, the government re-sells his inventories. After time T, the economy is back in steady state. We will need the following Lemma:

Lemma 9 (The time to unload inventories)**.** *Let $\Delta T \equiv T - T_\rho$. Then, as ω goes to zero:*

$$\Delta T^2 = \frac{2}{\alpha\frac{r+\kappa}{\sigma}}(1-e^{-\alpha T_\rho})\omega + o(\omega), \tag{49}$$

where $o(\omega)$ is a function such that $o(\omega)/\omega$ goes to zero as ω goes to zero, uniformly in T_ρ.

We prove the Lemma in Section A.9.1. After the recovery, the government unloads its inventories at a speed guaranteeing that the price grows at rate r or, equivalently, that ξ grows at rate $r+\kappa$. That is, we have $\xi(t) = \bar{\xi}e^{-(r+\kappa)(T-t)}$. Because of iso-elastic utilities, this immediately implies that investors scale up their holdings by a factor $e^{\frac{r+\kappa}{\sigma}(T_2-t)}$. Thus, the *change* in welfare induced by the intervention is equal to:

$$\begin{aligned}
&\frac{\alpha}{r+\alpha}\int_{T_\rho}^{T}\sum_i \pi_i\hat{\bar{\varepsilon}}_i\frac{a_i^{1-\sigma}}{1-\sigma}\left(e^{\frac{(1-\sigma)(r+\kappa)}{\sigma}(T-t)}-1\right)e^{-rt}\,dt \\
= \;&\frac{\alpha}{r+\alpha}e^{-rT_2}\sum_i \pi_i\hat{\bar{\varepsilon}}_i\frac{a_i^{1-\sigma}}{1-\sigma}\left(\frac{1}{r+\frac{(1-\sigma)(r+\kappa)}{\sigma}}\left[e^{\left(r+\frac{(1-\sigma)(r+\kappa)}{\sigma}\right)(T-T_\rho)}-1\right]-\frac{1}{r}\left[e^{r(T-T_\rho)}-1\right]\right) \\
= \;&\frac{\alpha}{r+\alpha}e^{-r(T_\rho+\Delta T)}\sum_i \pi_i\hat{\bar{\varepsilon}}_i\frac{a_i^{1-\sigma}}{1-\sigma}\frac{(1-\sigma)(r+\kappa)}{2\sigma}\left(\Delta T^2 + o(\Delta T^2)\right) \qquad (50) \\
= \;&\frac{\alpha}{r+\alpha}e^{-rT_\rho}\sum_i \pi_i\hat{\bar{\varepsilon}}_i a_i^{1-\sigma}\frac{r+\kappa}{2\sigma}\left(\Delta T^2 + o(\Delta T^2)\right). \qquad (51)
\end{aligned}$$

To go from the second to the third line, we used the Taylor expansion:

$$\frac{1}{B}\left(e^{B\Delta}-1\right) = \Delta T + \frac{B}{2}\Delta T^2 + o(\Delta^2),$$

keeping in mind that, from Lemma 9, $\Delta T \to 0$ as $\omega \to 0$, uniformly in T_ρ – i.e. it will take very little time for the government to re-sell very little inventories. To go from the third to the fourth line, we canceled out the $1-\sigma$ and noted that $T = T_\rho + \Delta T = T_\rho + o(1)$. Now plugging the Taylor approximation (49) into (51), we find that the change in welfare is, then,

$$\omega \frac{1}{r+\alpha} \sum_i \pi_i \hat{\bar{\varepsilon}}_i a_i^{1-\sigma} e^{-rT_\rho} \left(1 - e^{-\alpha T_\rho}\right) + o(\omega). \tag{52}$$

Putting the two together. The next thing to do is to take expectations with respect to the random recovery time, T_ρ, in (48) and (52), and add up the two terms. This shows that the total expected change in welfare is equal to:

$$\begin{aligned}
& -\omega \frac{\alpha}{r+\alpha} \frac{1}{r} \left(1 - \frac{\rho}{r+\rho}\right) \sum_i \pi \hat{\bar{\varepsilon}}_i^C a_i^{C\,1-\sigma} + \omega \frac{1}{r+\alpha} \left(\frac{\rho}{r+\rho} - \frac{\rho}{r+\rho+\alpha}\right) \sum_i \pi_i \hat{\bar{\varepsilon}}_i a_i^{1-\sigma} + o(\omega) \\
\Leftrightarrow \quad & -\omega\alpha \frac{r+\alpha}{r+\rho} \sum_i \pi \hat{\bar{\varepsilon}}_i^C a_i^{C\,1-\sigma} + \omega \frac{r+\alpha}{r+\rho} \frac{\rho\alpha}{r+\rho+\alpha} \sum_i \pi_i \hat{\bar{\varepsilon}}_i a_i^{1-\sigma} + o(\omega) \\
\Leftrightarrow \quad & \frac{\omega\alpha}{(r+\rho)(r+\alpha)} \left[-\sum_i \pi_i \hat{\bar{\varepsilon}}_i^C a_i^{C\,1-\sigma} + \frac{\rho}{r+\rho+\alpha} \sum_i \pi_i \hat{\bar{\varepsilon}}_i a_i^{1-\sigma} \right] + o(\omega),
\end{aligned}$$

which is the formula of the proposition.

Verifying that dealers hold no inventories. During the recovery, dealers find it weakly optimal to hold no inventories because, by construction, the price grows at rate r. During the crisis, we need to verify that:

$$-(r+\kappa)\xi^C(t) + \dot{\xi}^C(t) + \rho \left(\xi(t \,|\, t) - \xi^C(t)\right) < 0. \tag{53}$$

Recall that, by construction of the government intervention, $\xi^C = (1-\omega)^{-\sigma} \bar{\xi}_0^C$ is constant over time. The price at the recovery time is, on the other hand:

$$\xi(t,t) = \bar{\xi} e^{-(r+\kappa)\Delta T}.$$

So (53) becomes

$$-(r+\kappa)(1-\omega)^{-\sigma} \bar{\xi}_0^C + \rho \left(\bar{\xi} e^{-(r+\kappa)\Delta T} - (1-\omega)^{-\sigma} \bar{\xi}_0^C\right) < 0.$$

Next, note that $\xi(t \,|\, t)$ is decreasing. Indeed if the crisis lasts longer, the government holds more inventories, and ΔT will be larger, meaning that it will take longer to unwind these inventories after the recovery. Thus, in order for (53) to hold at all times during the crisis, it is necessary and sufficient that it holds at time zero, i.e.

$$-(r+\kappa)(1-\omega)^{-\sigma} \bar{\xi}_0^C + \rho \left(\bar{\xi} - (1-\omega)^{-\sigma} \bar{\xi}_0^C\right) < 0.$$

But we restrict attention to economies such that this condition holds with strict inequality when $\omega = 0$. Therefore, by continuity, it also holds with strict inequality if ω is close enough to zero.

A.9.1 Proof of Lemma 9

The time to unload inventories, ΔT. We start by deriving a simple equation for ΔT. Using equation (16), we know that T solves:

$$A_g(T_\rho) + \alpha \int_{T_\rho}^T e^{\alpha s} \left[A - \sum_{i=1}^I \pi_i U_i'^{-1} \left[\xi(s)\right] \right] ds = 0.$$

With the functional form $u_i(a) = \varepsilon_i a^{1-\sigma}/(1-\sigma)$ we have $U_i'^{-1}\left[\xi(s)\right] = \left[\bar{\varepsilon}_i/\xi(s)\right]^{1/\sigma}$ and $\xi(s) = \bar{\xi} e^{-(r+\kappa)(T-s)}$. Hence,

$$A_g(T_\rho) + \alpha \int_{T_\rho}^T e^{\alpha s} \left[A - \sum_{i=1}^I \pi_i \left[\frac{\bar{\varepsilon}_i}{\bar{\xi}}\right]^{\frac{1}{\sigma}} e^{\left(\frac{r+\kappa}{\sigma}\right)(T-s)} \right] ds = 0.$$

Notice steady-state market clearing after the recovery implies that $\sum_{i=1}^I \pi_i \left(\bar{\varepsilon}_i/\bar{\xi}\right)^{1/\sigma} = A$. Thus, after some calculations, and letting $\gamma \equiv \frac{r+\kappa}{\sigma}$, we arrive at

$$\frac{A_g(T_\rho)-A}{A}+\frac{\gamma}{\gamma-\alpha}e^{\alpha\Delta T}-\frac{\alpha}{\gamma-\alpha}e^{\gamma\Delta T}=0 \Leftrightarrow -1+\omega\left(1-e^{-\alpha T_\rho}\right)+\frac{\gamma e^{\alpha\Delta T}-\alpha e^{\gamma\Delta T}}{\gamma-\alpha}=0$$

which can be written $F\left(\omega\left(1-e^{-\alpha T_\rho}\right),\Delta T^2\right)=0$, where

$$F(z,u)\equiv -1+z+\frac{\gamma e^{\alpha\sqrt{u}}-\alpha e^{\gamma\sqrt{u}}}{\gamma-\alpha}.$$

It is then straightforward to show that, for each $z \geq 0$, $F(z,u)=0$ has a unique solution.

The Taylor expansion. Clearly, $F(0,0)=0$, i.e. when there are no inventories at time T_ρ ($z=0$) it takes not time to unload them ($u=0$). Next, we would like to know how u varies with ω. To do so, we need to apply the implicit function theorem (IFT). Clearly, $F(z,u)$ is continuously differentiable for all $z \geq 0$ and $u>0$. We need to show that this property is also true when $u=0$. First, we show that $F(z,u)$ is differentiable by calculating:

$$\begin{aligned}\frac{F(z,u)-F(z,0)}{u} &= \frac{1}{u}\left[\frac{\gamma e^{\alpha\sqrt{u}}-\alpha e^{\gamma\sqrt{u}}}{(\gamma-\alpha)}-1\right]\\ &= \frac{1}{u}\left[\frac{\gamma\left(1+\alpha\sqrt{u}+\frac{\alpha^2}{2}u+o(u)\right)-\alpha\left(1+\gamma\sqrt{u}+\frac{\gamma^2}{2}u+o(u)\right)}{\gamma-\alpha}-1\right]\\ &= \frac{1}{u}\left[\frac{\gamma-\alpha-\frac{\alpha\gamma}{2}(\gamma-\alpha)u+o(u)}{\gamma-\alpha}-1\right]\\ &= \frac{-\frac{\alpha\gamma}{2}u+o(u)}{u}\to -\frac{\alpha\gamma}{2},\end{aligned}$$

as $u\to 0$. Thus, $F(z,u)$ is differentiable at $(z,0)$, for all z, with $\partial F/\partial u(0,0)=-(\alpha\gamma)/2$. Next we show that $\partial F/\partial u$ is continuous at $(z,0)$. To see this note that

$$\frac{\partial F}{\partial u} = \frac{\alpha\gamma}{2(\gamma-\alpha)\sqrt{u}}\left(e^{\alpha\sqrt{u}}-e^{\gamma\sqrt{u}}\right)=\frac{\alpha\gamma}{2(\gamma-\alpha)\sqrt{u}}\left(1+\alpha\sqrt{u}-1-\gamma\sqrt{u}+o(\sqrt{u})\right)=-\frac{\alpha\gamma}{2}\left(1+o(1)\right),$$

so that $\partial F/\partial u \to -(\alpha\gamma)/2$ as $u\to 0$.

Now let $U(z)$ denote the solution of $F(z,u)=0$. An application of the IFT shows that for z small, we have:

$$U(z)=\frac{2}{\alpha\gamma}z+k(z).$$

where $k(z)$ is such that $k(z)/z$ goes to zero as z goes to zero. In the case at hand, we are looking at some recovery occurring a time T_ρ, when the government's inventory position is $A_g(T_\rho)=\omega(1-e^{-\alpha T_1})$. The above calculations show that

$$\Delta T^2=U(A_g(T_\rho))=\frac{2}{\alpha\gamma}\omega\left(1-e^{-\alpha T_\rho}\right)+k\left(\omega\left(1-e^{-\alpha T_\rho}\right)\right).$$

The last thing we need to argue is that $k\left(\omega\left(1-e^{-\alpha T_\rho}\right)\right)$ is $o(\omega)$, uniformly in T_ρ. To do so, we recall that for all $\varepsilon>0$, there is some $\eta>0$ such that $|z|<\eta$ implies that $|k(z)/z|<\varepsilon$. But $|\omega|<\eta$ implies that $|\omega\left(1-e^{-\alpha T_\rho}\right)|<\eta$ as well and therefore that:

$$\left|\frac{k\left(\omega\left(1-e^{-\alpha T_\rho}\right)\right)}{\omega}\right| \leq \left|\frac{k\left(\omega\left(1-e^{-\alpha T_\rho}\right)\right)}{\omega\left(1-e^{\alpha T_\rho}\right)}\right| \leq \varepsilon.$$

Thus $k\left(\omega\left(1-e^{-\alpha T_\rho}\right)\right)/\omega$ converges to zero uniformly in T_ρ, and we are done.

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